Treedepth & bounded expansion redux

(with lots of colors!)

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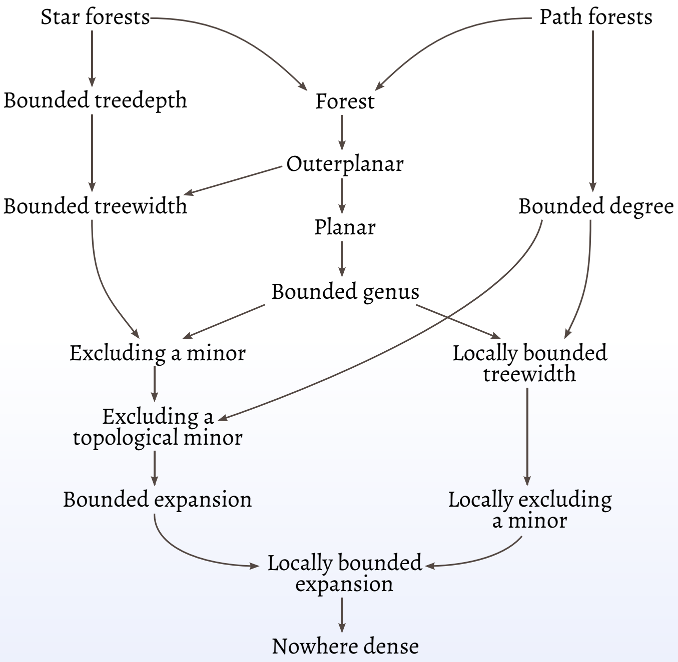
Theoretical Computer Science

RWTH AACHEN

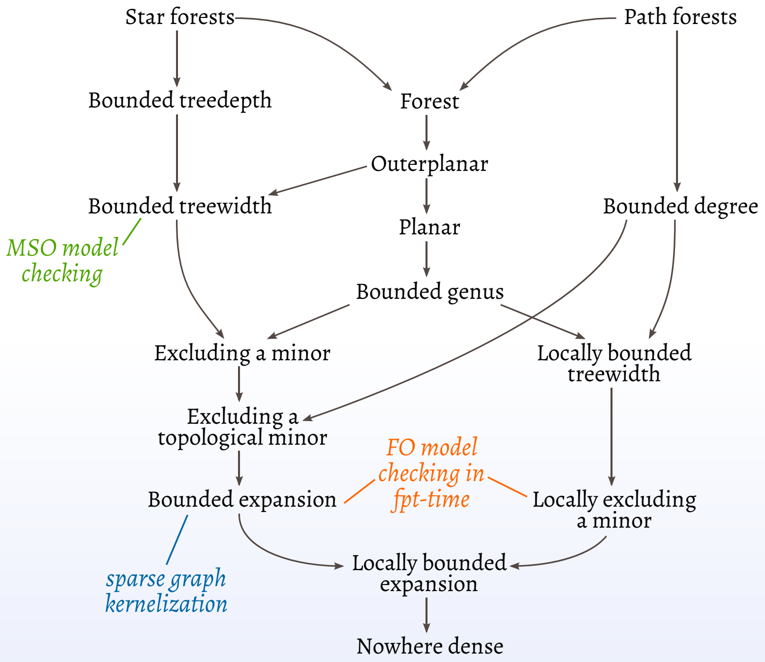
@Bergen 2013

Contents

The big picture



Star forests
Path forests
Bounded treedepth
Forest
Outerplanar
Bounded treewidth
Bounded degree
Planar
Bounded genus
Excluding a minor
Locally bounded treewidth
Excluding a topological minor
Bounded expansion
Locally excluding a minor
Locally bounded expansion
Nowhere dense



Star forests
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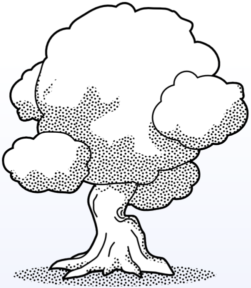
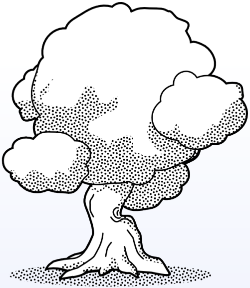
MSO model checking
Bounded genus
Excluding a minor
Locally bounded treewidth
Excluding a topological minor
FO model checking in fpt-time
Bounded expansion
Locally excluding a minor
sparse graph kernelization
Locally bounded expansion
Nowhere dense

Treedepth



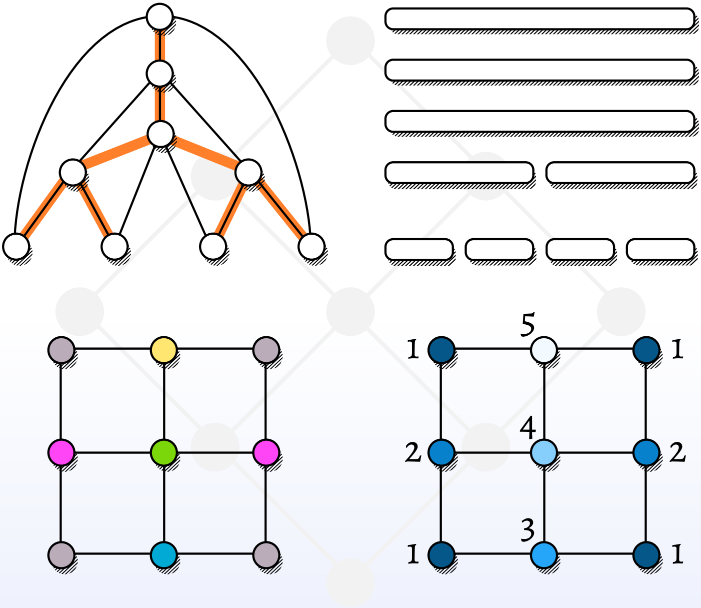


A strange width measure...

> "Why should this be useful?"
>
> —common first reaction to treedepth

A graph G has *treedepth* at most d if

- G is a subgraph the closure of a tree (forest) of height $\leq d$
- G has a *centered coloring* with d colors
- G has a *ranked coloring* with d colors
- G is the subgraph of a *trivially perfect graph* with clique size at most d


1
5
1
2
4
2
1
3
1

Interesting tidbits about treedepth

- Treedepth is subgraph-closed
- $\textbf{tw}(G) \leq \textbf{pw}(G) \leq \textbf{td}(G) - 1$
- Maximum path length is $2^{\textbf{td}(G)} - 1$

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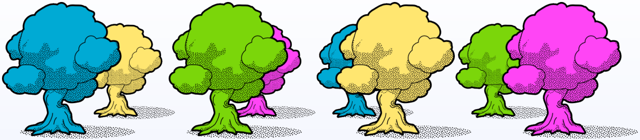
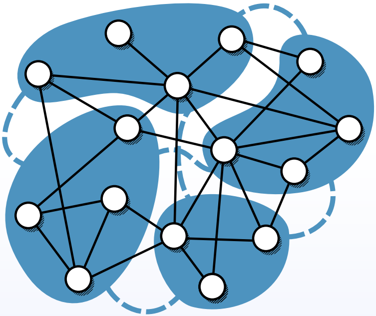
$\Rightarrow$ a DFS is a treedepth-decomposition of depth $\leq 2^{\textbf{td}(G)} - 1$

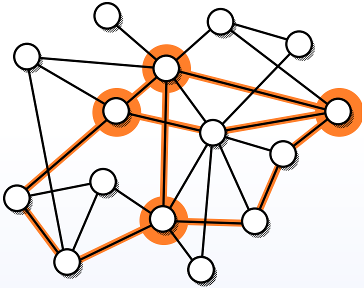
- Minor-closed property (thus in fpt) and MSO-expressible (thus in fpt, again)
- Graphs of bounded treedepth are WQO under the induced subgraph relation (Even true if one allows a finite set of vertex labels)

Graph classes of bounded expansion

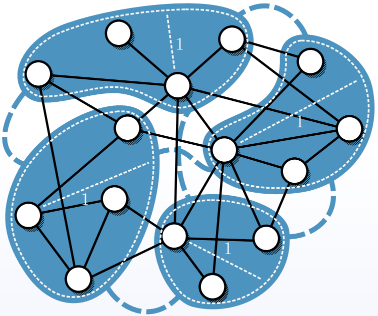


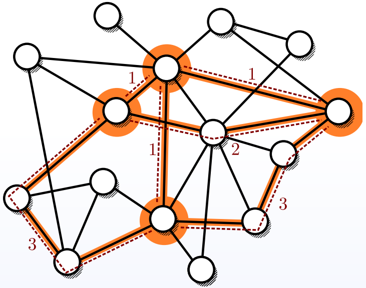
Minors, top-minors





Shallow minors, top-minors





Bounded expansion

For a graph G we denote by $G \triangledown r$ the set of its r-shallow minors.

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Definition (Grad, Expansion)

For a graph G, the *greatest reduced average density* is defined as

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A graph class $\mathcal{G}$ has *bounded expansion* if there exists a function f such that $\nabla_r(\mathcal{G}) \leq f(r)$ for all $r \in \mathbf{N}$.

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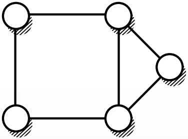

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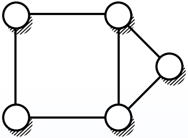
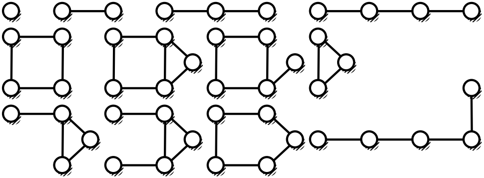


$G \triangledown 0$



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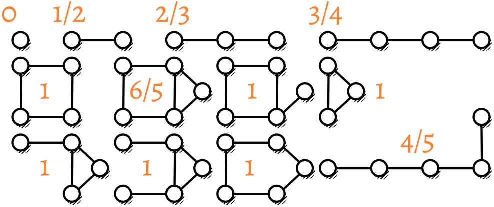

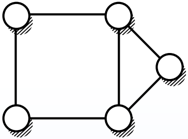

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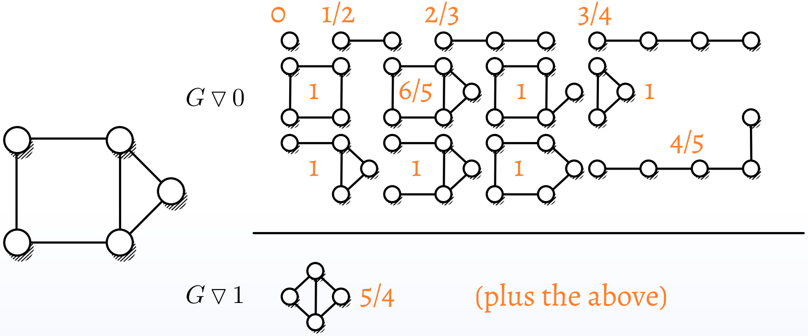

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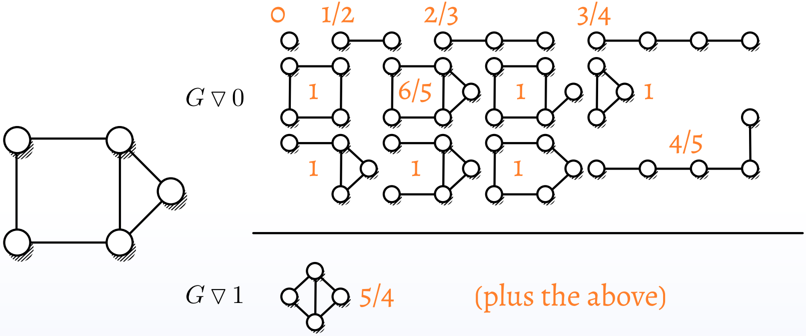

Note that $G \triangledown 0 \subseteq G \triangledown 1 \subseteq \ldots$

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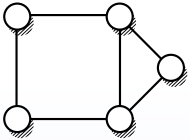

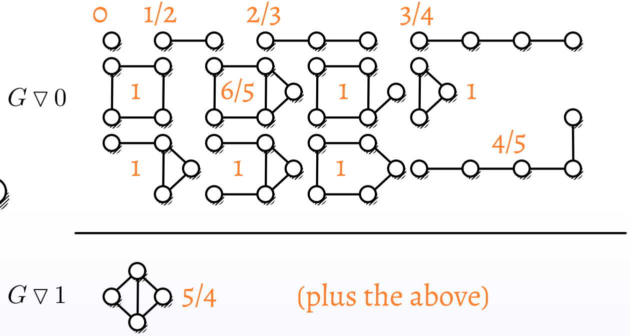

Note that $G \triangledown 0 \subseteq G \triangledown 1 \subseteq \ldots$

$\nabla_0(G) = 1.2$

$\nabla_1(G) = 1.25$

$\nabla_t(G) = 1.25, t \geq 1$

Graph classes with bounded expansion

Class	$f(r)$
Planar	3
H-minor-free	$O(\lvert H\rvert \log \lvert H\rvert)$
H-top-minor-free	$O(\lvert H\rvert^2)$
d-regular	$d(d-1)^{r-1}$
Crossing number c	$O(\sqrt{cr})$
$\mathcal{G}(n, p/n)$	a.a.s with some $g(r, p)$

Alternative characterizations

A graph class $\mathcal{C}$ has bounded expansion if there exists a function f such that for each $G \in \mathcal{C}$ and each $r \in \mathbf{N}$

- $\tilde{\nabla}_r(G) \leq f(r)$ (Using top-shallow-minors)
- G admits an r-centered-coloring with $\leq f(r)$ colors
- G admits a r-treedepth-coloring with $\leq f(r)$ colors
- G has a linear ordering such that the number of weakly-r-accessible vertices is $\leq f(r)$
- For each orientation $\vec{G}_0$ of G with $\Delta^-(\vec{G}_0) \leq f(0)$ there exists a *transitive fraternal augmentation*

$$\vec{G}_0 \subseteq \vec{G}_1 \subseteq \vec{G}_2 \ldots$$

such that $\Delta^-(\vec{G}_i) \leq f(i)$

Top-grad

Grads can also be defined via shallow *topological* minors:

$$\tilde{\nabla}_r(G) = \max_{H \in G \tilde{\triangledown} r} \frac{|E(H)|}{|V(H)|}$$

where $G \tilde{\triangledown} r$ denotes the set of all r-shallow top minors.

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Grad and Topgrad are related as follows:

$$\tilde{\nabla}_r(G) \leq \nabla_r(G) \leq 4(4\tilde{\nabla}_r(G))^{(r+1)^2}$$

r-centered-coloring

Vertex-coloring of the graph G such that every connected subgraph $H \subseteq G$

- either receives more than r colors
- at least one color appears exactly once in H

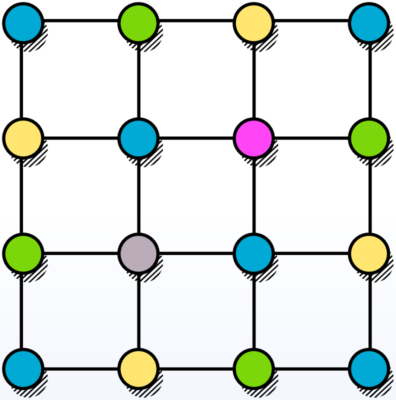
Graphs classes of bounded expansion are exactly those classes whose members need only $\chi_r(G) < f(r)$ colors. In particular

$$\nabla_r(G) \leq (2r+1)\binom{\chi_{2r+2}(G)}{2r+2}$$

$$\chi_r(G) \leq poly(\tilde{\nabla}_{2^{r-2}+1/2}(G))$$

where the degree of the polynomial is roughly 2^{2^r}

r-centered-coloring

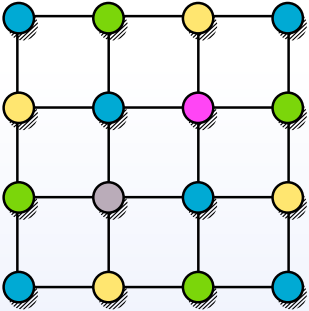


3-centered coloring of a grid

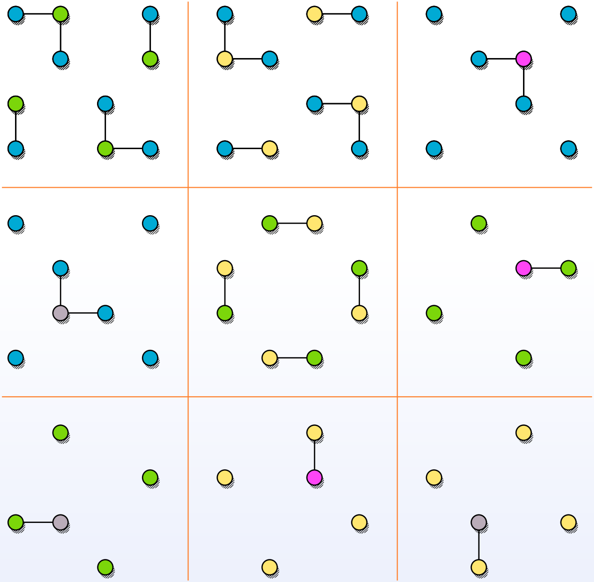
r-treedepth-coloring

Vertex-coloring of the graph G such that every subgraph induced by $i < r$ colors classes has treedepth at most i.

An r-centered coloring is also an r-treedepth-coloring!



r-treedepth-coloring



Transitive fraternal augmentations

How to calculate r-centered coloring of a graph G whose expansion is bounded by f?

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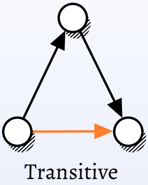
How to calculate r-centered coloring of a graph G whose expansion is bounded by f?

- Create orientation $\vec{G}_0$ of G such that $\Delta^-(\vec{G}_0) \leq f(0)$
- For $1 \leq i \leq r$
  - $G_i := G_{i-1}$
  - Add *transitive* edges to G_i
  - Add *fraternal* edges to G_i such that the fraternal edges alone are acyclic and the in-degree is minimized



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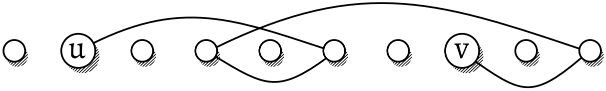
One can show that $\Delta^-(\vec{G}_i) \leq f(i)$, i.e. the coloring number of the graphs does not increase too much. A proper coloring of $\vec{G}_{O(r \log r)}$ yields an r-centered coloring for G.

Weak coloring number

Consider linear ordering of the vertices:



u is *weakly-r-accessible* from v if $u < v$ and there exists a u-v-path of length at most r whose leftmost vertex is u.

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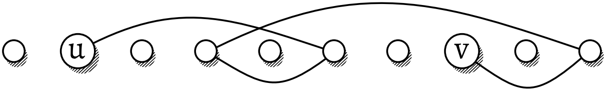

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Define $B_r(v)$ as the set of all weakly-r-accessible vertices. Graph classes of bounded expansion are exactly those classes whose members G satisfy

$$wcol_r(G) = \min_{\pi \in S_{|G|}} \max_{v \in G} B_r^{\pi}(v) \leq f(r)$$

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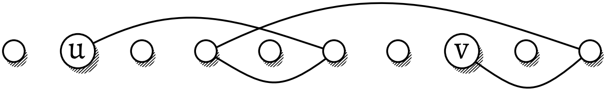

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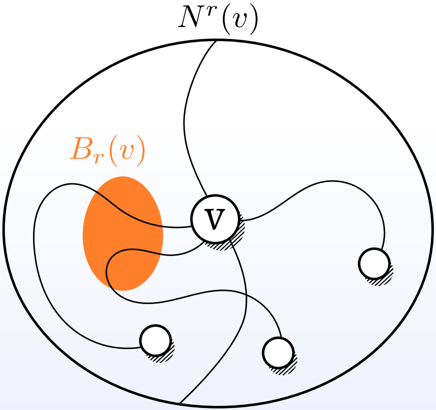
In particular,

$$\nabla_{\frac{r-1}{2}}(G) + 1 \leq wcol_r(G) \leq poly(\nabla_{\frac{r-1}{2}}(G))$$

Weak coloring number

$B_r(v)$ is the set of all vertices left of v which can be reached from v by a path of length r using nothing left of the target vertex.



Alternative alternative characterizations

> "So many choices"
>
> —Dr. Dre

A graph class $\mathcal{C}$ has bounded expansion if there exists a function f such that for each $G \in \mathcal{C}$ and each $r \in \mathbf{N}$

- $\chi(G \triangledown r) \leq f(r)$
- G admits a r-treewidth-coloring with $\leq f(r)$ colors

A graph class $\mathcal{C}$ has bounded expansion if

- there exists a constant c and a *strongly topological, monotone, degree bound* graph parameter α such that $\mathcal{C} \subseteq \{G \mid \alpha(G) \leq c\}$
- ...

Algorithms

Dvořák's Algorithm

- Constant-factor approximation for t-Dominating Set
- Constant c depends on t, expansion
- Outputs t-dominating set D and $2t+1$-scattered set $S \subseteq D$ such that $|D| \leq c \cdot |S|$
- Since for any optimal DS D^* it holds that

$$|S| \leq |D^*| \leq |D| \leq c \cdot |S|$$

the set $|D|$ has quality ratio c

Input: A graph G

Calculate an ordering W of G with bounded $wcol_t$;
$D \leftarrow \emptyset$;
$S' \leftarrow \emptyset$;
$R \leftarrow V(G)$;

while $R \neq \emptyset$ **do**
  Let $v \in R$ be the next vertex according to W;
  $S' \leftarrow S' \cup \{v\}$;
  $D \leftarrow D \cup \{v\} \cup B_{2t+1}(v)$;
  Remove every vertex t-dominated by D from R;

Output S', D;

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From the algorithm it is apparent that $|D| = O(|S'|)$ and that D is a dominating set. But S' is not necessarily $2t+1$-scattered.

Making it scattered

Construct $2t+1$-scattered set $S \subseteq S'$ with $S' = O(S)$ as follows

- Create auxiliary graph $H = (S', E')$ where $xy \in E'$ if $d_G(x, y) \leq 2t + 1$

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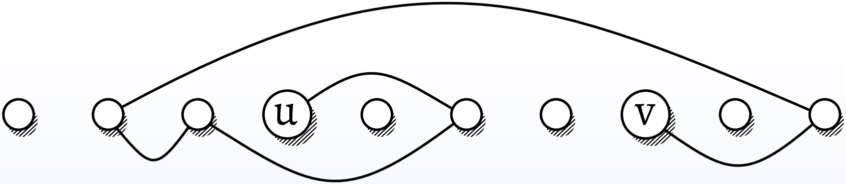
$\Rightarrow$ Color with $c'+1$ colors and pick largest color class as S

Prove that v has only a constant number of back-neighbours in H using the original vertex ordering.

- $uv \in E'$, i.e. $d_G(u, v) \le 2t + 1$



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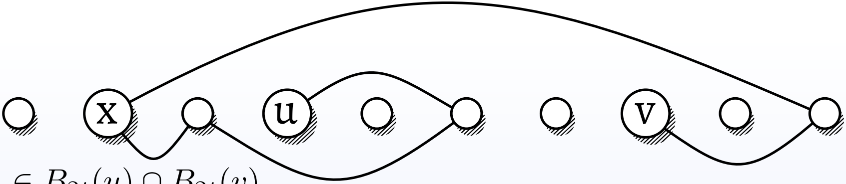

$\in B_{2t}(u) \cap B_{2t}(v)$

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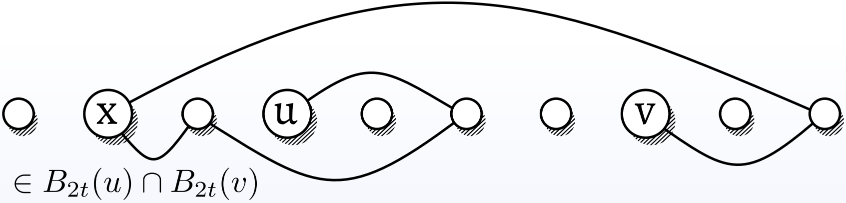
- $x \in B_{2t}(v)$ and $|B_{2t}(v)|$ is a constant!



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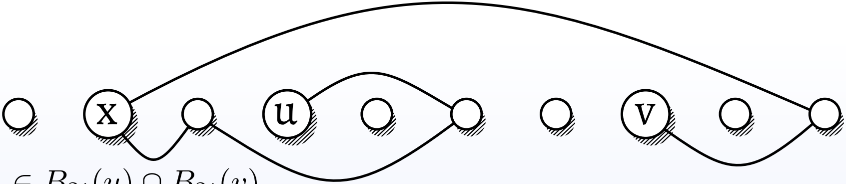
- $x \in B_{2t}(v)$ and $|B_{2t}(v)|$ is a constant!

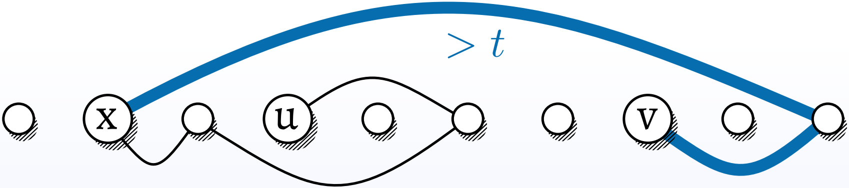
$\Rightarrow$ Show that x cannot be "shared" with other back-neighbour of v



$\in B_{2t}(u) \cap B_{2t}(v)$

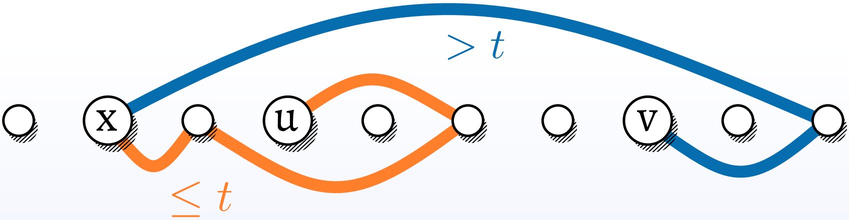
Prove that x cannot lie on any path of length $2t + 1$ from v to another vertex $u' < v$.

- $d_G(x, v) > t$, otherwise x would dominate v and thus $v \notin S'$



Prove that x cannot lie on any path of length $2t+1$ from v to another vertex $u' < v$.

- $d_G(x, v) > t$ implies $d_G(x, u) \leq t$



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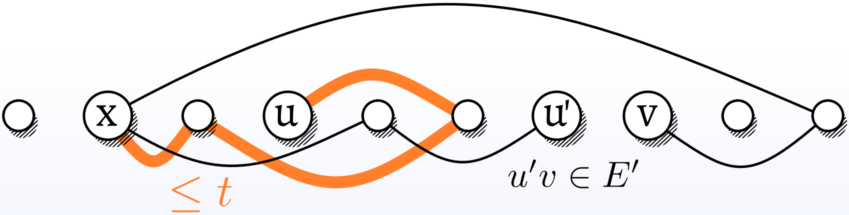

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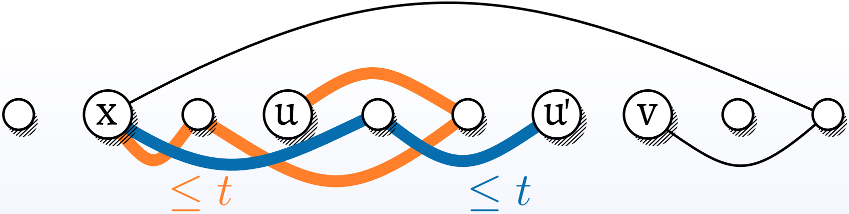

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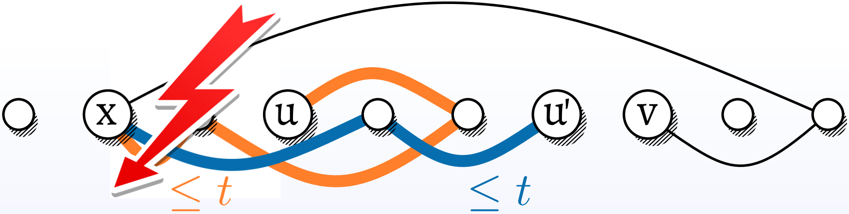

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Observe that for vertices x, y with $d_G(x, y) = t$, these vertices have distance at most 2 in the t-th transitive fraternal augmentation $\vec{G}_t$.

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We can easily track the distances these edges bridge! Therefore we can answer distance-queries for vertex pairs at distance $\leq t$ correctly in constant time by consulting an annotated version of $\vec{G}_t$. (We can also correctly conclude for all other pairs that they are further than t apart)

First Order Model checking, light

Theorem (Nešetřil, Ossana de Mendez)

Let $\mathcal{C}$ be a class of bounded expansion and p a fixed integer. Let ϕ be first-order sentence. Then there exists a linear-time algorithm to check $\exists X : |X| \leq p \wedge G[X] \models \phi$

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Uses: check whether a fixed graph H is a subgraph / induced subgraph of another graph G in linear time.

First Order Model checking, hardcore

Theorem (Dvořák, Kral, Thomas ’10, Grohe, Kreutzer ’11)

Let $\mathcal{C}$ be a class of bounded expansion and ϕ a FO sentence. For $G \in \mathcal{C}$ one can decide in linear time whether $G \models \phi$.

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- Many variants of local search can be expressed in FO

$\Rightarrow$ Improvement steps in fpt-time with linear dependence on input size

Kernelization

Continuation of previous meta-results on planar, bounded genus, H-minor-free and H-topological minor-free graphs.

- First problem: natural parameters too strong for many problems, e.g. a linear kernel for FEEDBACK VERTEX SET would imply the same for general graphs which in turn implies coNP $\subseteq$ NP/poly (important trick: subdividing the edges of a graph $|G|$ times yields a graph with low grad!)

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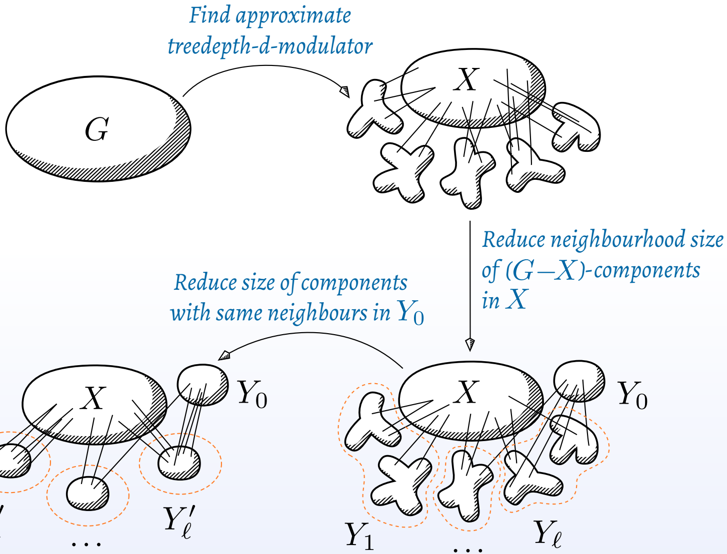
- Solves first problem: not closed under edge subdivision
- Solves second problem: WQO of bounded-treedepth graphs means we can replace protrusions by one of their subgraphs (We thus restrict ourselves to hereditary graph classes)

Proof sketch



Conclusion

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