

Practise what you preach:  
sparsity in the real world.



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[felix.reidl@gmail.com](mailto:felix.reidl@gmail.com)

ASSG'17

# Part I

Theory vs Practise

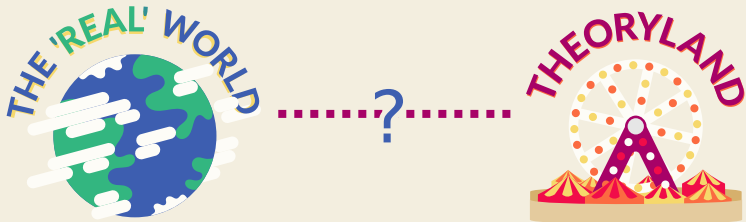


# A bit of (hard) introspection

“About ten years ago, some computer scientists came by and said they heard we have some really cool problems. They showed that the problems are NP-complete and went away!”

–Joseph Felsenstein in 1997

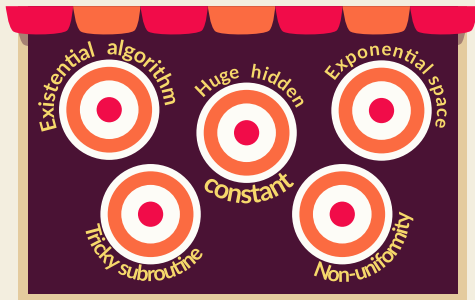
Downey RG, Fellows MR, Stege U. Computational tractability: The view from mars. Bulletin of the EATCS. 1999 Oct;69:73-97.



# Our reputation



**Small “tricks”**





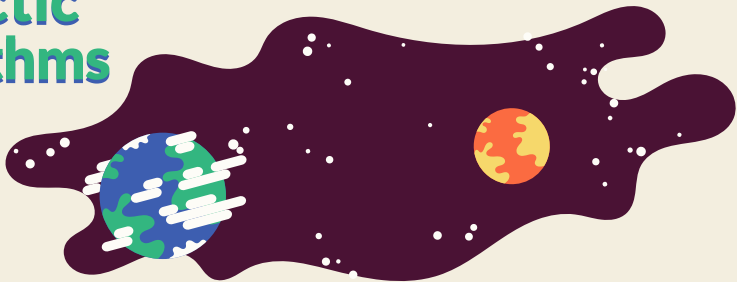
# Our reputation



Small “tricks”



Galactic Algorithms



# A communication problem

“Practical algorithm”

Polynomial time

Linear time

Short description

Logspace

Reduces to  
well-known problem

Easy to understand



# A communication problem

“Practical algorithm”

Polynomial time

Linear time

Short description

Logspace

Reduces to  
well-known problem

Easy to understand

“I just cloned it from  
github and it ran fine  
on my data set.”



# The long and winding road



Theory only

Pseudocode

Implementable

No tricks

Executable

Github  
(or similar)

Usable

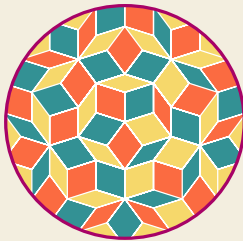


# The long and winding ~~road~~



## ***Part II***

# Background(s)

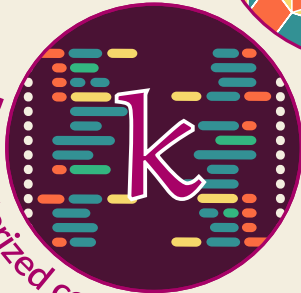


# Three domains

Structural sparseness



Parameterized complexity

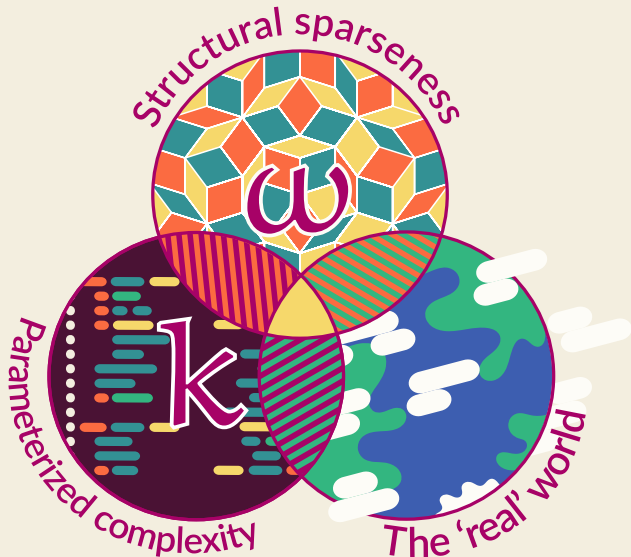


Parameterized complexity



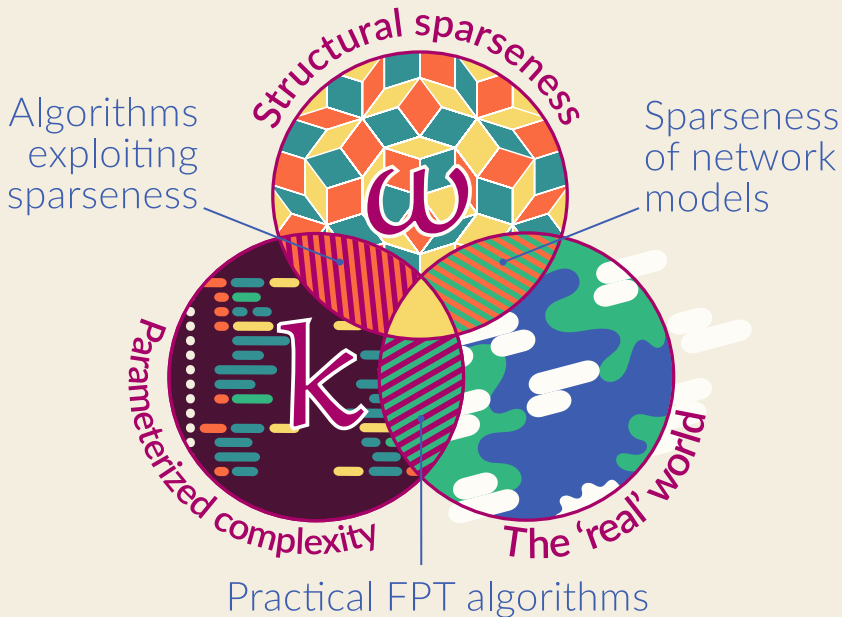
The 'real' world

# Four intersections

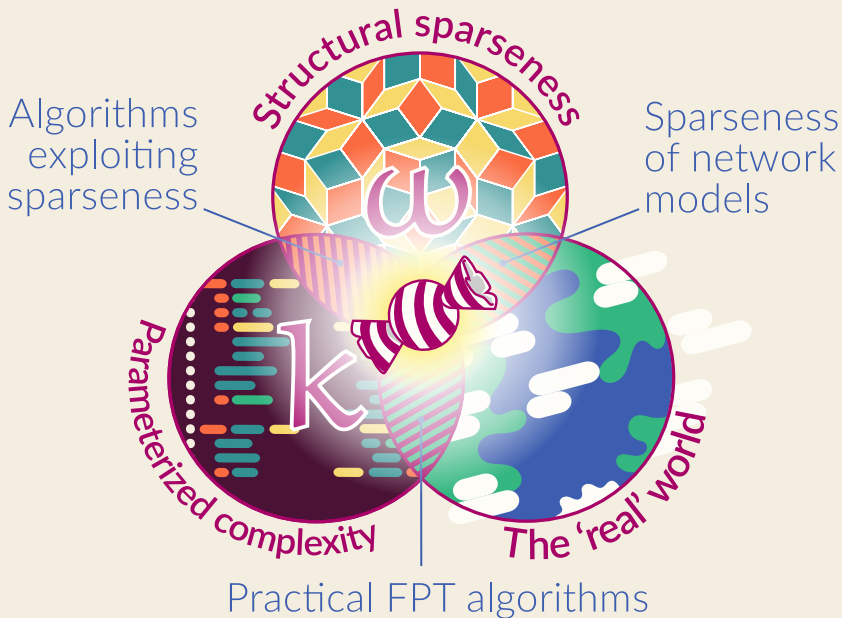




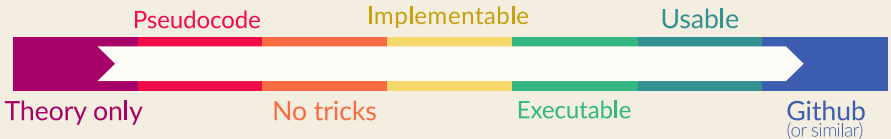
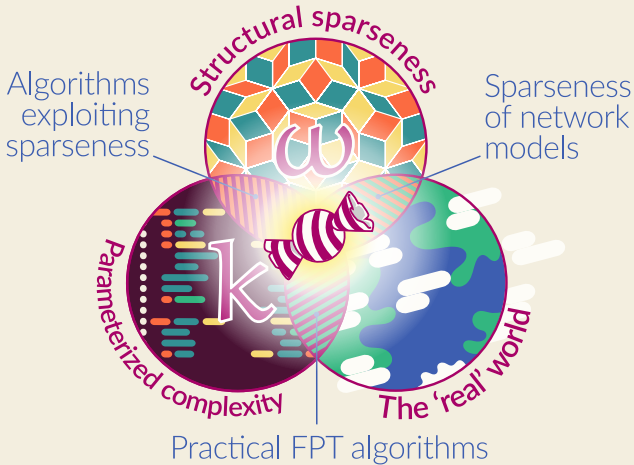
# Four intersections



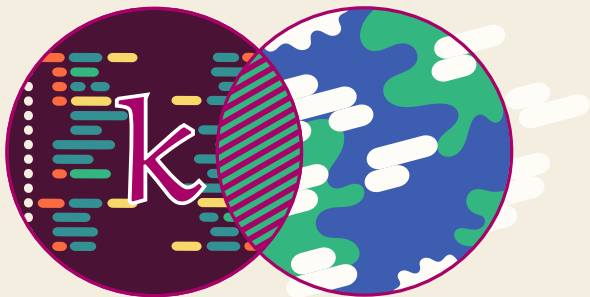
# Four intersections



# Four intersections



# FPT $\cap$ real world



Criticism of **trickery** in algorithms (ask Mike about the “PTAS industry”) was central in proposing parameterized complexity as an alternative<sup>1</sup>.

Questions of practicality are encoded in the DNA of the field, and it shows!

<sup>1</sup>Downey R. **Parameterized complexity for the skeptic.**

InProc. 18th IEEE Annual Conference on Computational Complexity 2003 Jul 7 (Vol. 132).

# FPT $\cap$ real world

“The collection of methods for classifying problems as fixed-parameter tractable, for designing FPT algorithms, for designing better FPT algorithms and **transferring these results to practical implementations**, [...] has developed with surprising vigor.”

Fellows MR. **Blow-ups, win/win's, and crown rules: Some new directions in FPT.**  
In International Workshop on Graph-Theoretic Concepts in Computer Science 2003  
Jun 19 (pp. 1-12). Springer, Berlin, Heidelberg.

Questions of practicality are **raised regularly**, see e.g.

Cheetham J, Dehne F, Rau-Chaplin A, Stege U, Taillon PJ.  
**Solving large FPT problems on coarse-grained parallel machines.**  
Journal of Computer and System Sciences. 2003 Dec 31;67(4):691-706.

Langston M. **Practical FPT implementations and applications.**  
In International Workshop on Parameterized and Exact Computation 2004  
Sep 14 (pp. 291-291). Springer, Berlin, Heidelberg.

Abu-Khazam FN, Cai S, Egan J, Shaw P, Wang K.  
**Turbo-Charging Dominating Set with an FPT Subroutine: Further Improvements and Experimental Analysis.**  
In International Conference on Theory and Applications of Models of Computation 2017 Apr 20 (pp. 59-70). Springer, Cham.



# FPT $\cap$ real world

While many FPT algorithms are impractical (either because of **trickery** or parameters that are **not small in practise**), there is a consensus in the field that **practicality matters**.

Parameterized complexity therefore not only gives us a rich theoretical framework, we also find many past attempts of applying said framework to real world problems.



## Takeaway:

Parameterized complexity offers the framework, the the institutions, and the culture to investigate the question of practicality.

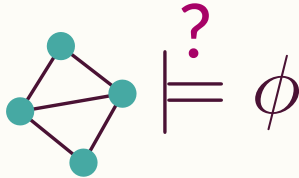


# Exhibit A

# SEQUOIA

MSO-model checking  
against all odds

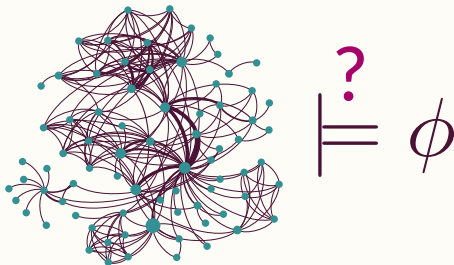
# MSO model checking



Given an  $\text{MSO}_2$  formula in the language of (annotated) graphs and a graph, test whether the graph is a model of the formula.



# MSO model checking



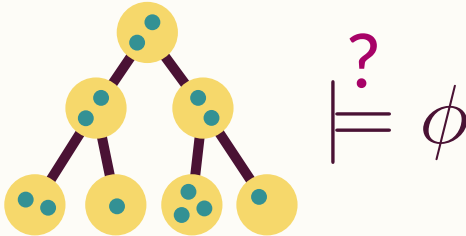
Given an  $\text{MSO}_2$  formula in the language of (annotated) graphs and a graph, test whether the graph is a model of the formula.

**Not FPT for graphs of moderately unbounded treewidth!**

Kreutzer S. **On the Parameterised Intractability of Monadic Second-Order Logic.**  
In CSL 2009 Sep 7 (Vol. 9, pp. 348-363).

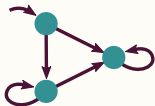
Kreutzer S, Tazari S. **Lower bounds for the complexity of monadic second-order logic.**  
In Logic in Computer Science (LICS), 2010 25th Annual IEEE Symposium  
on 2010 Jul 11 (pp. 189-198). IEEE.

# MSO model checking

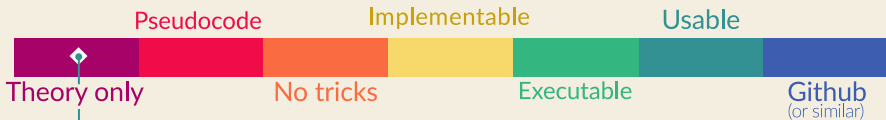


Given an  $\text{MSO}_2$  formula in the language of (annotated) graphs and a graph **of bounded treewidth**, test whether the graph is a model of the formula.

# Engineering Courcelle's Theorem



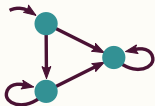
State  
explosion



Courcelle B. The monadic second-order logic of graphs.  
I. Recognizable sets of finite graphs.

Information and computation.  
1990 Mar 1;85(1):12-75.

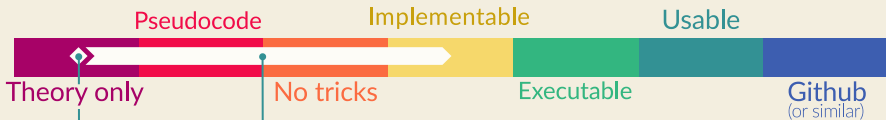
# Engineering Courcelle's Theorem



State  
explosion



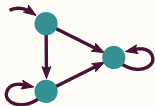
DP using  
Hintikka games



Theory only  
Courcelle B. The monadic  
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1990 Mar 1;85(1):12-75.

Kneis J, Langer A, Rossmanith P.  
**Courcelle's theorem—a game-theoretic approach.**  
Discrete Optimization. 2011 Nov 30;8(4):568-94.

# Engineering Courcelle's Theorem



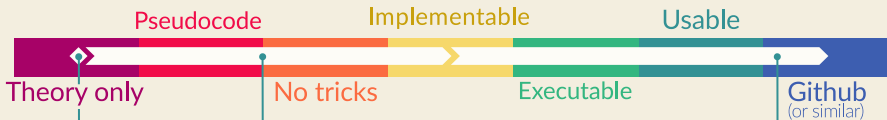
State  
explosion



DP using  
Hintikka games



Software Engineering  
& optimization

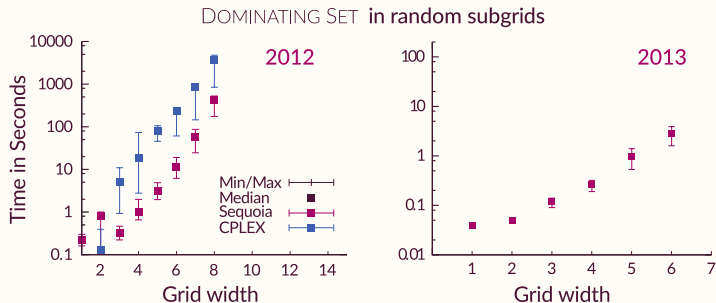


Courcelle B. **The monadic second-order logic of graphs. I. Recognizable sets of finite graphs.** Information and computation. 1990 Mar 1;85(1):12-75.

Langer A. **Fast algorithms for decomposable graphs.** (Doctoral dissertation, Dissertation, Aachen, Techn. Hochsch., 2013).  
<https://github.com/sequoia-mso/sequoia-core>

Kneis J, Langer A, Rossmanith P.  
**Courcelle's theorem—a game-theoretic approach.**  
Discrete Optimization. 2011 Nov 30;8(4):568-94.

# Sequoia performance



<b>2013</b>	CONNECTED DOMINATING SET			Sequoia		CPLEX		
	Instance	n	tw	time	solution	time	solution	gap
	Hannover (small)	673	8	3min	319	—	327	41%
	Hannover (large)	956	9	9min	376	—	385	42%
	Berlin	2599	11	3h 12min	1269	—	1342	35%

Langer A. **Fast algorithms for decomposable graphs.**

(Doctoral dissertation, Dissertation, Aachen, Techn. Hochsch., 2013).

Langer A, Reidl F, Rossmanith P, Sikdar S. **Evaluation of an MSO-solver.**

In 2012 Proceedings of the Fourteenth Workshop on Algorithm Engineering and Experiments (ALENEX) 2012 Jan 16 (pp. 55-63). Society for Industrial and Applied Mathematics.

# Exhibit B

# Toboggan

High-throughput FPT

# Flow decomposition

AGGACGTAGATAGCTAGCTAATGCTACGATCAGAGGACGTAGATTTATTACCAT

TACCGAATACGAACTAGGATATCGATCGATCAGAGGCCCAATAGGGAATATCCG

TACCGAATACGAACTAGGATATCGATCGATTGATCTATAATAGTAGAATATCCG

Shared segments between DNA/RNA strands create ambiguity in the assembly problem.

Hartman T, Hassidim A, Kaplan H, Raz D, Segalov M. **How to split a flow?**. In INFOCOM, 2012 Proceedings IEEE 2012 Mar 25 (pp. 828-836). IEEE.

Tomescu AI, Kuosmanen A, Rizzi R, Mäkinen V.

**A novel min-cost flow method for estimating transcript expression with RNA-Seq.** BMC bioinformatics. 2013 Apr 10;14(5):S15.



# Flow decomposition

AGGACGTAG	ATAGCTAGC	TAATGCTAC	GATCAGAGG	ACGTAGATT	TATTACCAT
TACCGAATA	CGAACTAGG	ATATCGATC	GATCAGAGG	CCCAATAGG	GAATATCCG
TACCGAATA	CGAACTAGG	ATATCGATC	GATTGATCT	ATAATAGTA	GAATATCCG

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**A novel min-cost flow method for estimating transcript expression with RNA-Seq.** BMC bioinformatics. 2013 Apr 10;14(5):S15.

# Flow decomposition



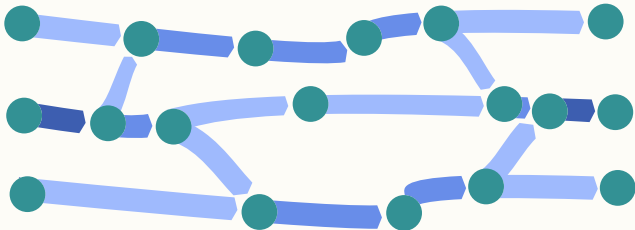
Shared segments between DNA/RNA strands create ambiguity in the assembly problem.

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**A novel min-cost flow method for estimating transcript expression with RNA-Seq.** BMC bioinformatics. 2013 Apr 10;14(5):S15.

# Flow decomposition



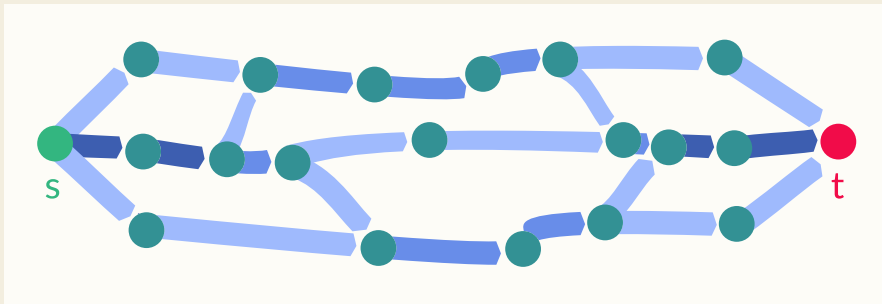
Connecting overlapping segments ('k-mers') and counting their frequencies ('abundance') yields a DAG and a flow.

Hartman T, Hassidim A, Kaplan H, Raz D, Segalov M. **How to split a flow?**. In: INFOCOM, 2012 Proceedings IEEE 2012 Mar 25 (pp. 828-836). IEEE.

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# Flow decomposition



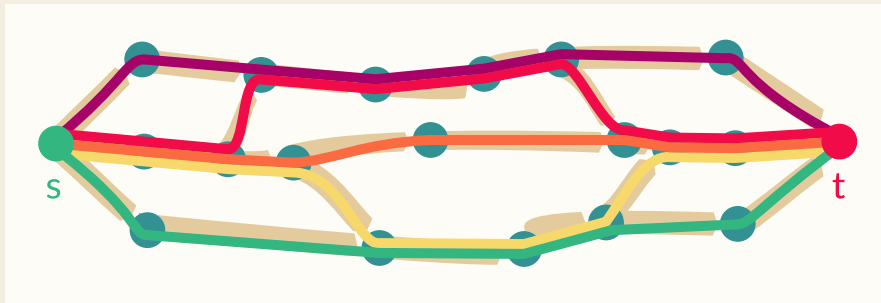
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Tomescu AI, Kuosmanen A, Rizzi R, Mäkinen V.

**A novel min-cost flow method for estimating transcript expression with RNA-Seq.** BMC bioinformatics. 2013 Apr 10;14(5):S15.

# Flow decomposition



Our task is to split the flow into  $k$  **weighted** s-t-paths in order to recover the original DNA/RNA strands.

Hartman T, Hassidim A, Kaplan H, Raz D, Segalov M. **How to split a flow?**. In INFOCOM, 2012 Proceedings IEEE 2012 Mar 25 (pp. 828-836). IEEE.

Tomescu AI, Kuosmanen A, Rizzi R, Mäkinen V.

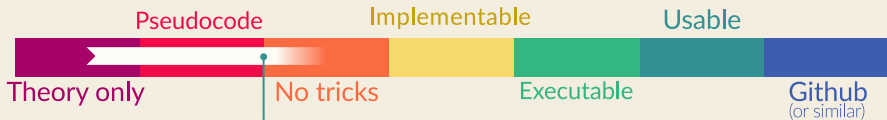
**A novel min-cost flow method for estimating transcript expression with RNA-Seq.** BMC bioinformatics. 2013 Apr 10;14(5):S15.

# Engineering flow decomposition

$$2^{O(k^2)} n$$

FPT algorithm

No poly kernel



Kloster K, Kunke P, O'Brien MP, Reidl F,  
Sánchez Villaamil F, Sullivan BD, van der Poel A.

**A practical fpt algorithm for Flow Decomposition and transcript assembly.**

arXiv preprint arXiv:1706.07851. 2017 Jun 23.

# Engineering flow decomposition

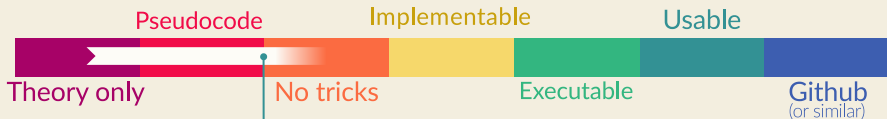
$$2^{O(k^2)} n$$

FPT algorithm



DP using ILPs\*  
to encode  
constraints

No poly kernel



Kloster K, Kunke P, O'Brien MP, Reidl F,  
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\*More precisely: systems of Diophantine equations



# Engineering flow decomposition

$$2^{O(k^2)} n$$

FPT algorithm

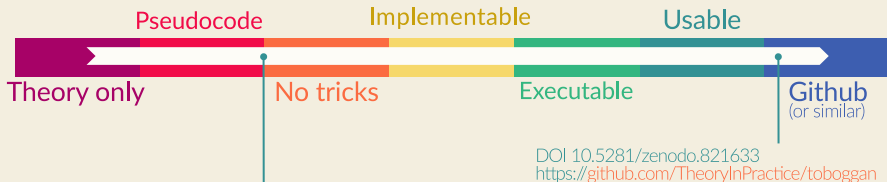
No poly kernel



DP using ILPs\*  
to encode  
constraints



Preprocessing, early-out  
heuristics and optimization



Kloster K, Kunke P, O'Brien MP, Reidl F,  
Sánchez Villaamil F, Sullivan BD, van der Poel A.

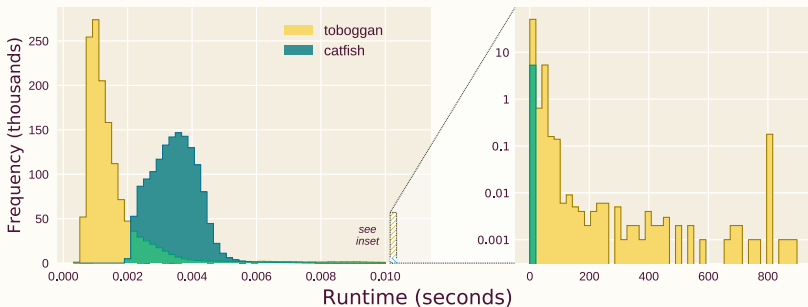
**A practical fpt algorithm for Flow Decomposition and transcript assembly.**

arXiv preprint arXiv:1706.07851. 2017 Jun 23.

\*More precisely: systems of Diophantine equations

# Toboggan vs Catfish

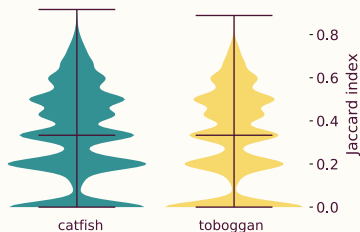
A comparison over ~4 million instances



Groundtruth recovery

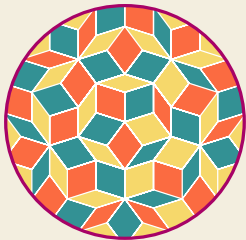
k	instances	Catfish	Toboggan
2	63.2791 %	0.992	<b>0.995</b>
3	22.0775 %	0.967	<b>0.969</b>
4	8.5237 %	<b>0.931</b>	0.930
5	3.4920 %	0.886	0.886
6	1.5375 %	<b>0.830</b>	0.828
7	0.6698 %	<b>0.788</b>	0.780
8	0.2889 %	<b>0.767</b>	0.766
9	0.1241 %	0.740	<b>0.743</b>
10	0.0070 %	0.752	<b>0.802</b>
11	0.0004 %	0.500	0.500
All	100 %	0.973	<b>0.975</b>

Groundtruth similarity



## ***Part III***

Once more,  
*with sparsity*



# Elevator pitch: structural sparseness

A **graph measure** is an isomorphism invariant function that maps graphs to  $\mathbb{R}^+$

e.g. density, average degree, clique number, degeneracy treewidth, etc.

A **parameterised graph measure** is a family of graph measures  $(f_r)_{r \in \mathbb{N}_0}$ .

A graph class  $\mathcal{G}$  is  **$f_r$ -bounded** if there exists  $g$  s.t.

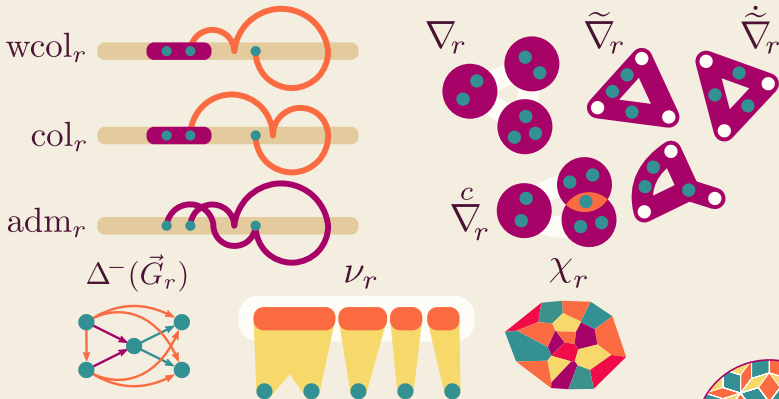
$$f_r(\mathcal{G}) = \sup_{G \in \mathcal{G}} f_r(G) \leq g(r) \text{ for all } r$$



# Elevator pitch: structural sparseness

Nešetřil & Ossona de Mendez:

Many notions of  $f_r$ -boundedness are equivalent!



We call classes in which these measures are bounded classes of bounded expansion

# Sparsity $\cap$ FPT



In “Fixed-Parameter Tractability and Completeness”<sup>1</sup>, the first technique Mike and Rod put of forward is the  $O(n^3)$  minor-testing routine by Robertson and Seymour.

If practicality in the DNA of the field, the graph minor theorem is its stem cell.

<sup>1</sup>Downey RG, Fellows MR. **Fixed-parameter tractability and completeness.** Cornell University, Mathematical Sciences Institute; 1992.

# Sparsity $\cap$ FPT: big hammers

MSO-Model checking in graphs of bounded treewidth

Arnborg S, Lagergren J, Seese D. **Easy problems for tree-decomposable graphs.**

Journal of Algorithms. 1991 Jun 1;12(2):308-40.

FO-Model checking in nowhere-dense graphs

Grohe M, Kreutzer S, Siebertz S. **Deciding first-order properties of nowhere dense graphs.**

Journal of the ACM (JACM). 2017 Jun 16;64(3):17.

Meta-Kernelization in sparse classes

Bodlaender HL, Fomin FV, Lokshtanov D, Penninkx E, Saurabh S, Thilikos DM. **(Meta) kernelization.**

In Foundations of Computer Science, 2009. FOCS'09. 50th Annual IEEE Symposium on

2009 Oct 25 (pp. 629-638). IEEE.

Fomin FV, Lokshtanov D, Saurabh S, Thilikos DM. **Bidimensionality and kernels.**

In Proceedings of the twenty-first annual ACM-SIAM symposium on Discrete Algorithms 2010

Jan 17 (pp. 503-510). Society for Industrial and Applied Mathematics.

Gajarský J, Hliněný P, Obdržálek J, Ordyniak S, Reidl F, Rossmanith P, Sánchez Villaamil F,

Sikdar S. **Kernelization using structural parameters on sparse graph classes.**

Journal of Computer and System Sciences. 2017 Mar 31;84:219-42.

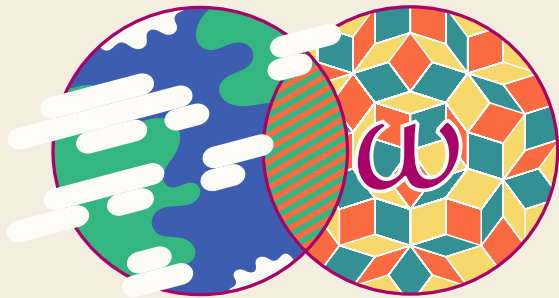
Kim EJ, Langer A, Paul C, Reidl F, Rossmanith P, Sau I, Sikdar S. **Linear**

**kernels and single-exponential algorithms via protrusion decompositions.**

ACM Transactions on Algorithms (TALG). 2016 Feb 12;12(2):21.



# Real world $n$ sparsity

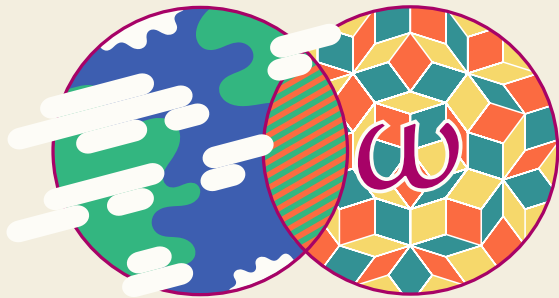


The big question:  
Are real-world graphs (*networks*) **sparse**?

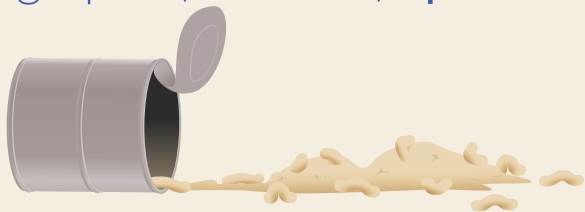




# Real world $n$ sparsity

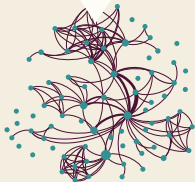


The big question:  
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# Graph classes vs. networks

Network instances



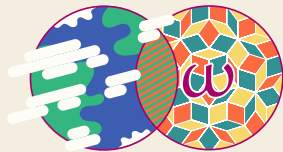
???



Mathematical Theory

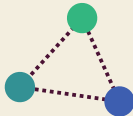
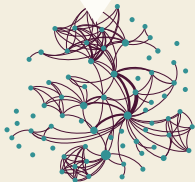
Theorem.

$$Pr[\|G\| \geq \xi k] \leq \left( \frac{e\beta D^2}{2n\xi k e^{D^2/2n}} \right)^{\xi k}$$



# Graph classes vs. networks

Network instances

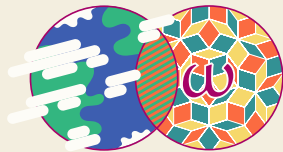


Network model

Mathematical Theory

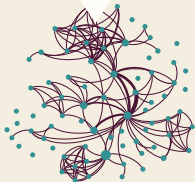
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# Graph classes vs. networks

Network instances

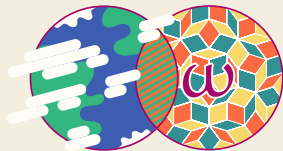
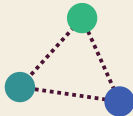


Mathematical Theory

Theorem.

$$\Pr[\|G\| \geq \xi k] \leq \left( \frac{e\beta D^2}{2n\xi k e^{D^2/2n}} \right)^{\xi k}$$

Network model



# Structurally sparse random graphs

Sparse Erdős–Rényi graphs are structurally sparse

Nešetřil J, de Mendez PO, Wood DR.

**Characterisations and examples of graph classes with bounded expansion.**

European Journal of Combinatorics. 2012 Apr 30;33(3):350-73.

Random graphs with fixed degree distribution that have quickly vanishing tails are structurally sparse

Demaine ED, Reidl F, Rossmanith P, Sánchez Villaamil F, Sikdar S, Sullivan BD.

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arXiv preprint arXiv:1406.2587. 2014 Jun 10.

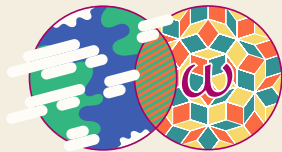
Random intersection graphs (in certain regimes) are structurally sparse.

Farrell M, Goodrich TD, Lemons N, Reidl F, Sánchez Villaamil F, Sullivan BD.

**Hyperbolicity, degeneracy, and expansion of random intersection graphs.**

International Workshop on Algorithms and Models for the Web-Graph 2015

Dec 10 (pp. 29-41). Springer, Cham.



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Are these models in these regimes representative of real world graphs?



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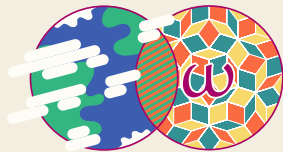
Farrell M, Goodrich TD, Lemons N, Reidl F, Sanchez Villaamil F, Sullivan BD.

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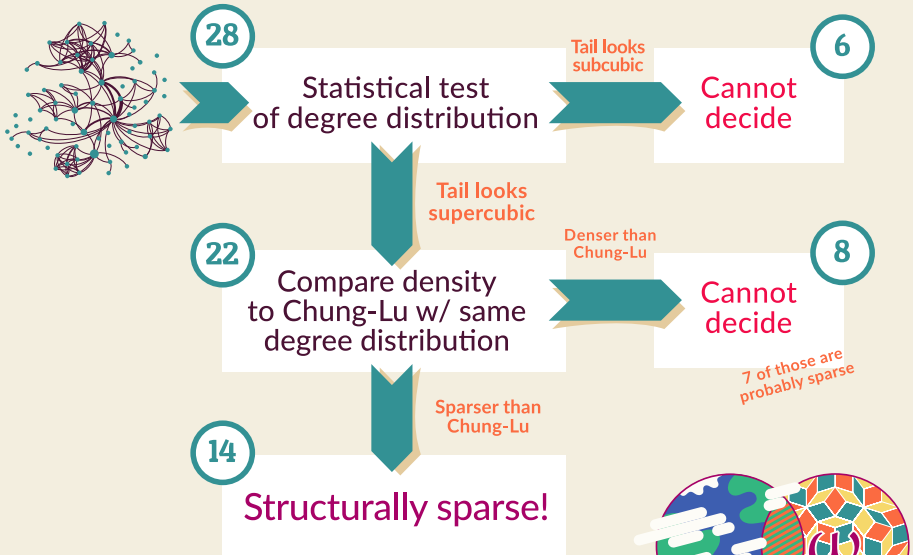
International Workshop on Algorithms and Models for the Web-Graph 2015

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Are these models in these regimes representative of real world graphs?



# Real structural sparseness



Reidl F. *Structural sparseness and complex networks*.  
(Doctoral dissertation, Dissertation, Aachen, Techn. Hochsch., 2015).





# Real world $\cap$ sparsity

All experiments so far as well as properties of several important network models point towards complex networks being structurally sparse.

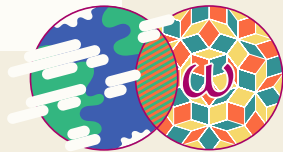
Some of these results directly contradict some widely held assumptions about networks!

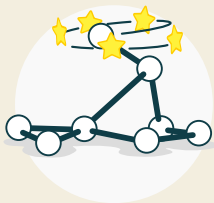
There are good reasons to dismiss these assumptions.  
Don't ask me about that unless you want a half-hour rant.



**Takeaway:**

Many real-world networks are structurally sparse.





# Exhibit C

# CONCUSS

Combatting Network Complexity  
Using Structural Sparsity

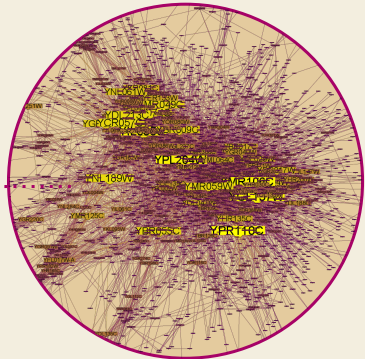
Motif counting using  
low-treedepth colourings

# Motif-counting

We want to count the number of times a given motif graph



appears in a larger host graph.



Motifs that appear more often than expected probably play an important role in the network

Milo R, Shen-Orr S, Itzkovitz S, Kashtan N, Chklovskii D, Alon U.  
**Network motifs: simple building blocks of complex networks.**  
Science. 2002 Oct 25;298(5594):824-7.

Ribeiro P, Silva F, Kaiser M. **Strategies for network motifs discovery.**  
InE-Science, 2009. e-Science'09. Fifth IEEE International Conference on 2009 Dec 9 (pp. 80-87). IEEE.



# Engineering motif-counting

$$f(h) \cdot 2^{O(h^2)} n$$

$$f(h) \cdot h^{O(h)} n$$



Nešetřil J, De Mendez PO.

**Sparsity: graphs, structures, and algorithms.**

Springer Science & Business Media; 2012 Apr 24.

Demaine ED, Reidl F, Rossmanith P,  
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**Structural sparsity of complex networks:**

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# Engineering motif-counting

tf-augmentations



Absolutely  
impractical

dtf-augmentations



Test colouring  
after each step



Nešetřil J, De Mendez PO.

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Reidl F. **Structural sparseness  
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(Doctoral dissertation, Dissertation,  
Aachen, Techn. Hochsch., 2015).

# Engineering motif-counting

tf-augmentations



Absolutely  
impractical

dtf-augmentations



Test colouring  
after each step



Good engineering &  
heuristic improvements

Pseudocode

Implementable

Usable

Theory only

No tricks

Executable

Github  
(or similar)

Nešetřil J, De Mendez PO.

**Sparsity: graphs, structures, and algorithms.**

Springer Science & Business Media; 2012 Apr 24.

Reidl F. **Structural sparseness  
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(Doctoral dissertation, Dissertation,  
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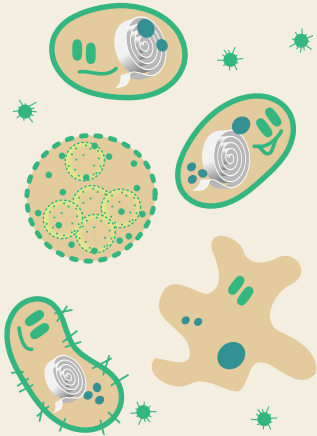
**Theory in Practice Group** (NCSU)  
with great help from students  
Clayton G. Hobbs & Brandon Mork  
<https://github.com/TheoryInPractice/CONCUSS>

# Exhibit D

# CATLAS

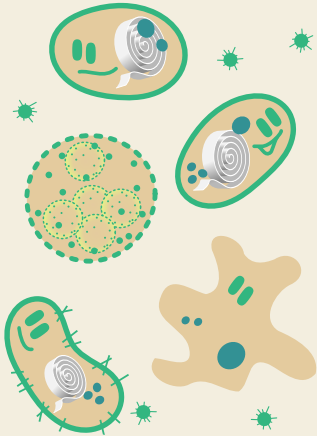
Metagenome exploration  
using hierarchical domination  
of de-Bruijn graphs

# Metagenomics

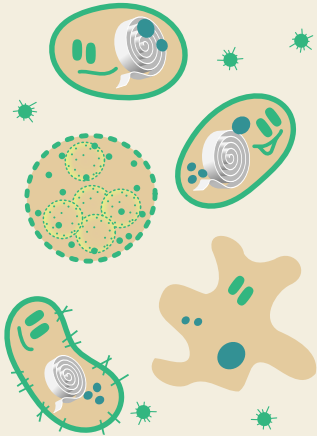




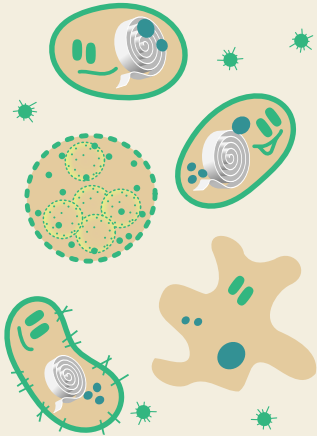
# Metagenomics



# Metagenomics



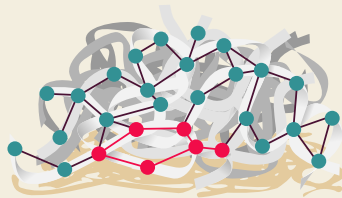
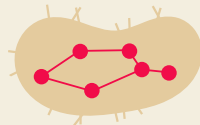
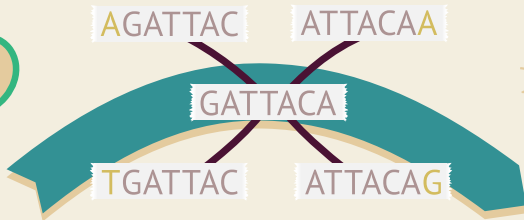
# Metagenomics



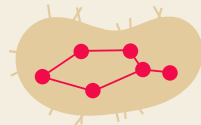
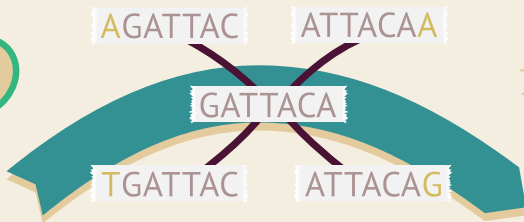
# De-Bruijn graphs



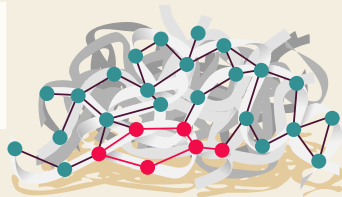
# De-Bruijn graphs



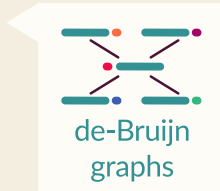
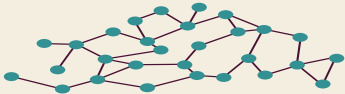
# De-Bruijn graphs



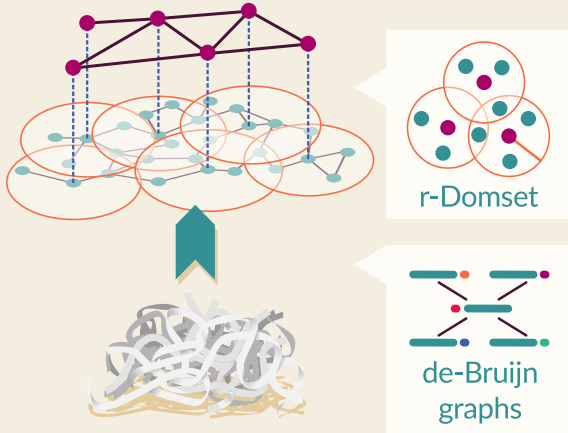
Bounded  
degree



# CATLAS Overview



# CATLAS Overview





# CATLAS Overview



Minhash  
Sketches



Domset



r-Domset



de-Bruijn  
graphs



# CATLAS Overview



Minhash  
Sketches



Domset



r-DTFAs



r-Domset



Dvořák's  
Algorithm\*



de-Bruijn  
graphs

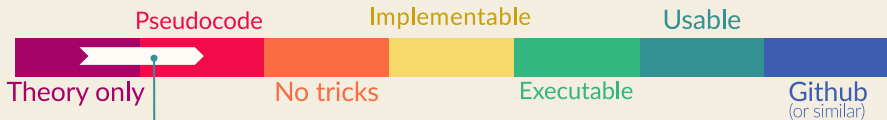


# Engineering Dvořák's algorithm



$wcol_{2r}$

Approximation is  
terrible in practise

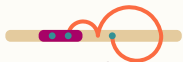


Dvořák Z. Constant-factor approximation  
of the domination number in sparse graphs.

European Journal of Combinatorics.

2013 Jul 31;34(5):833-40.

# Engineering Dvořák's algorithm



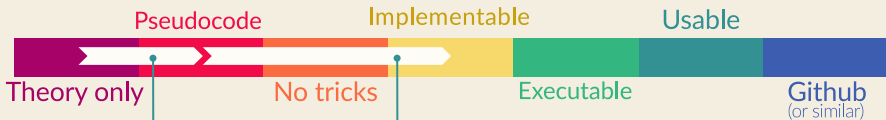
$$\text{wcol}_{2r}$$

Approximation is terrible in practise



$$\Delta^-(\vec{G}_{2r})$$

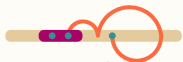
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# Engineering Dvořák's algorithm



$$wcol_{2r}$$

Approximation is terrible in practise



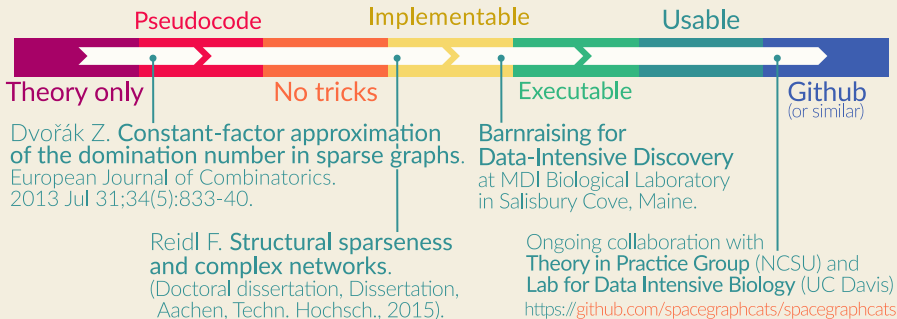
$$\Delta^-(\vec{G}_{2r})$$

Approximation is terrible in practise

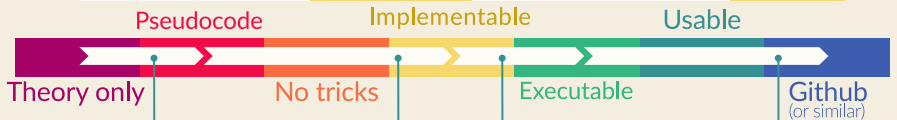
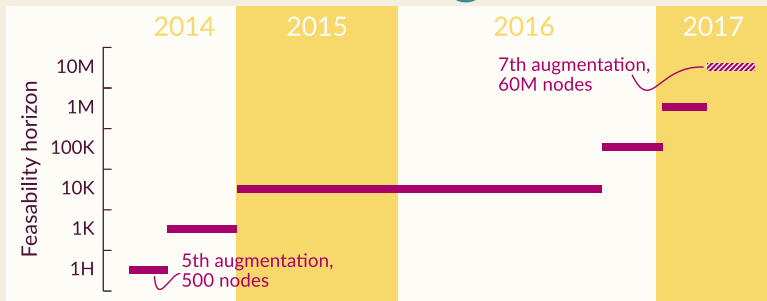


$$\Delta^-(\vec{G}_r)$$

Approximation is tunable (heuristic)



# Orders of magnitude



Dvořák Z. **Constant-factor approximation of the domination number in sparse graphs.**  
European Journal of Combinatorics.  
2013 Jul 31;34(5):833-40.

Reidl F. **Structural sparseness and complex networks.**  
(Doctoral dissertation, Dissertation, Aachen, Techn. Hochsch., 2015).

Barnraising for **Data-Intensive Discovery**  
at MDI Biological Laboratory  
in Salisbury Cove, Maine.

Ongoing collaboration with  
**Theory in Practice Group (NCSU)** and  
**Lab for Data Intensive Biology (UC Davis)**  
<https://github.com/spacegraphcats/spacegraphcats>

## *Part IV*

# Future research



# Graph measures

Measures often NP-hard even for very low  $r$

Muzi I, O'Brien MP, Reidl F, Sullivan BD. **Being even slightly shallow makes life hard.**  
Accepted at MFCS'17

Z. Dvořák. **Asymptotical Structure of Combinatorial Objects.**  
(PhD thesis, Charles University, Faculty of Mathematics and Physics, 2007).



Polynomial-time computable measure (for some  $r > 0$ )?

Find fast exact algorithms or (good) approximations

Find new bounded expansion characterisations

Further map out relationship between measures



Find 'typical' values of measures in networks

Design heuristic tailored to real-world instances

**Template:** Bodlaender's Algorithm, treewidth heuristics



# Meta-theorems & algorithms



Approximation in bounded expansion classes  
Polynomial-time algorithms in sparse classes  
Hand-design algorithms for selected problems

**Template:** FPT running time/kernel size races



FO model-checking 'without tricks'  
Identify suitable FO-fragment



Make logic approachable for normal folks  
Implement more algorithms (student projects!)

**Template:** Courcelle's Theorem vs. hand-crafted DP

# A sparse theory of density



A theory of dense graphs with underlying sparse structure

A matching algorithmic meta-theory

**Template:** Treewidth/Rankwidth, Sparsity programme, FO-model checking in nowhere dense graphs



Hand-crafted FPT algorithms in such graphs



Apply to biocomp problems in e.g protein-protein interaction networks

**Template:** None yet?

# Takeaway



The combination of structural sparseness and parameterized algorithms has **the potential to deliver practical algorithms.**



The challenge of making theoretical results applicable in practise usually generates **theoretical follow-up questions.**



Aiming for practicality imposes restraints (no tricks!), but that is **not necessarily a bad thing.**



# THANKS!

## Questions?

A big thanks to my co-authors:

Fernando Sánchez Villaamil

Somnath Sikdar

Peter Rossmanith

Blair D. Sullivan

Alexander Langer

Michael P. O'Brien

Jakub Gajarský

Pål Grønås Drange

Jan Obdržálek

Sebastian Ordyniak

Timothy Goodrich

Petr Hlinený

Andrew van der Poel

Erik D. Demaine

Eun Jung Kim

Magnus Wahlström

Christophe Paul

Gregory Gutin

Ignasi Sau

Michal Pilipczuk

Nathan Lemons

Philipp Kunke

Stephan Kreutzer

Saket Saurabh

Alex J. Chin

Fedor V. Fomin

Marcin Pilipczuk

Markus S. Dregi

Martin L. Demaine

Matthew Farrell

Aaron B. Adcock

Daniel Lokshtanov

Irene Muzi

Jan Dreier

Konstantinos Stavropoulos

Kyle Kloster

Sebastian Siebertz

Li-Hsuan Chen

M. S. Ramanujan

Meirav Zehavi

