Integer Linear Programming

Input: An integer linear program with k variables.

Parameter: k

Question: Does this ILP have a solution?

This Problem is fixed parameter tractable.

The running time is $f(k)n^{O(1)}$, but the $f(k)$ are painfully large.

Proof: very involved...

Feedback Vertex Set

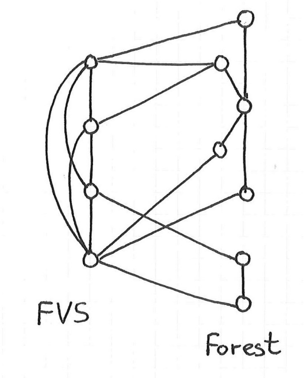
Input: A graph G and a number k

Parameter: k

Question: Are there $\leq k$ nodes whose removal makes G acyclic?

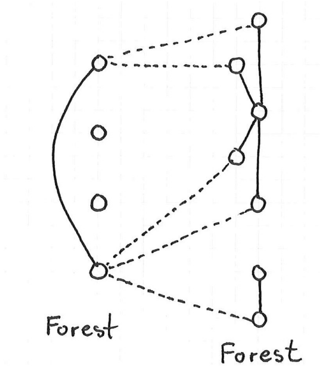
Is FVS fixed parameter tractable?

Iterative Compression



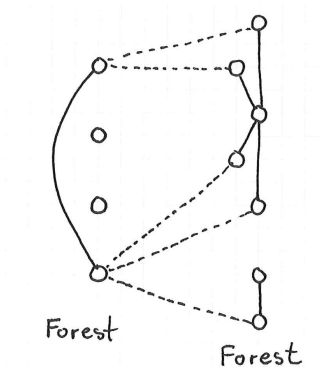
Assume we already know a FVS of size k.

Does this help to find a FVS of size $k-1$?



Step 1: Find a subset of the FVS to keep

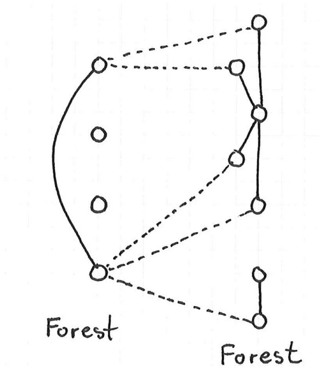
Plan: Add vertices from the forest to this FVS



Step 2: Apply reduction rules

Contract components of the FVS into one vertex

Contract degree-2 vertices in the forest



Step 3: Branching algorithm

If a leaf in the forest has two neighbors in the FVS:

a) put it into the FVS

b) delete it and decrease k

Running time

Size of the branching tree 4^k

Total size of all branching trees:

$$\sum_{j=0}^{k} \binom{k}{j} 4^j = 5^k$$

Total running time $5^k n^{O(1)}$

Overview

Introduction

Parameterized Algorithms

Further Techniques

Parameterized Complexity Theory

Advanced Techniques

Depth-First Search Trees

Input: A graph G and a number k

Parameter: k

Question: Is there a path of length k in G?

Construct a depth-first search tree.

What is the helpful property of a DFS tree?

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A Simple Theorem

Theorem

Let G be a graph and k a number.
Then it takes only polynomial time to find one of these:

1. *A cycle of length at least k*
2. *A tree decomposition of treewidth at most k*

Proof

$k+1$ cops slowly traverse the DFS tree

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Long Paths

The theorem allows us to find paths of length k easily:

1. If we find a cycle longer than k, there obviously is a path of length k as well
2. Otherwise we use the tree decomposition and Courcelle's theorem:
   $\exists x_1 \ldots \exists x_{k+1} (inc(x_1, x_2) \wedge \cdots \wedge inc(x_k, x_{k+1}) \wedge x_1 \neq x_2 \ldots)$

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Question: Can we solve Vertex Cover this way?

A Complicated Theorem

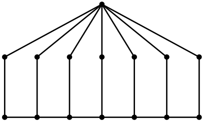
Theorem (Bodlaender)

Let G be a graph and k, l some numbers.
It takes $f(k, l)|G|$ steps to find one of these:

1. *A subdivision of the $2 \times k$ grid*
2. *A subdivision of the l-circus graph*
3. *A tree decompositon of G of treewidth $2(k-1)^2(l-1)+1$.*

Proof

Again, using a DFS tree.



7-circus graph

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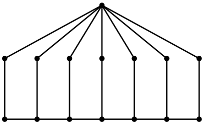

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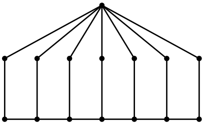

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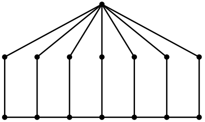
Why did it work for planar graphs?



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