Parameterized Algorithms WS 2021
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Exercise Sheet 07

Task T22

Consider the following variant of Vertex Cover:

Half Partial Vertex Cover

Input:	A graph $G = (V, E)$, an integer k.
Parameter:	The integer k.
Question:	Can k vertices in G cover at least $\lvert E\rvert/2$ edges?

Show that Half Partial Vertex Cover is in $W[1]$ by reducing it to STMA.

Task T23

Consider the following variant of Vertex Cover:

Partial Vertex Cover

Input:	A graph $G = (V, E)$, an integer k, and an integer t.
Parameter:	The integer k.
Question:	Can k vertices in G cover at least t edges?

Show that Partial Vertex Cover is W[1]-hard.

Task T24

In exercises T22 and T23 we saw that two different variants of the partial vertex cover problem are in W[1] or W[1]-hard respectively. Is one of the two variants W[1]-complete? Prove it!

Task H15 (7 credits)

Consider the following variant of Hitting Set:

Half 3-Hitting Set

Input:	A finite universe U, a family $\mathcal{F} \subseteq 2^U$ of sets of size exactly three, an integer k.
Parameter:	The integer k.
Question:	Can k elements of U hit at least $\lvert\mathcal{F}\rvert/2$ sets?

Show that Half 3-Hitting Set is in $W[1]$.

Task H16 (8 credits)

The Partial Vertex Cover problem is defined as follows: given a graph G and integers k and t, decide whether there exists k vertices that cover at least t edges. The parameter is the integer t (when parameterized by k only, the problem is W[1]-complete). The point of this exercise is to use color-coding to obtain a randomized FPT-algorithm for this problem.

1. Show that if $t \leq k$ then the problem is polynomial-time solvable. What happens if the maximum degree of the input graph is at least t?

2. Now use the following idea for coloring the vertices of the graph with two colors, say, green and red. Assume that there exists $S \subseteq V(G)$ of size at most k such that S covers at least t edges. Color each vertex red or green with probability 1/2. Show that the probability that vertices in S are colored green and all vertices in $\{u \in V(G) \setminus S \mid (u, v) \in E(G) \text{ for some } v \in S\}$ are colored red is a function of k and t. Call such a coloring a *proper coloring*.

3. Given a properly colored graph, we now need to identify a solution quickly. Note that the green vertices decompose the graph into connected components and that these contain the potential solution vertices. Show that in a properly colored graph, the solution is always the union of some green components, that is, the solution either includes all vertices of a green component or none. Hence any green component with k or more vertices that does not cover at least t edges can be discarded. Use this to design an algorithm that identifies a solution set in a proper two-colored graph.

4. Use all the above facts to design a randomized FPT-algorithm for the problem and analyze its time complexity.