RWTHAACHEN
UNIVERSITY

Exercise Sheet 04

Task T12

The problem 3-COLORABILITY is defined as follows: Given a graph G decide if it is possible to assign every node of G one of three colors, such that no two nodes with the same color are adjacent. This problem is fpt parameterized by the treewidth of the graph. Give an algorithm that solves the problem given a tree decomposition of width w in time $O(3^w \cdot w \cdot n)$.

Task T13

Design a fixed parameter algorithm for finding an $k \times k$-grid subgraph in a graph that is taken from a graph class with maximal degree d. The parameter is k. Analyse the running time.

Task T14

Give a parameterized algorithm that decides if a graph G contains k many vertex disjoint claws as induced subgraph. A claw is a $K_{1,3}$.

Task T15

Design a fixed parameter algorithm for finding a cycle of length *at least k* in an arbitrary graph G.

Task H8   (5 credits)

The problem DOMINATING SET is defined as follows: Given a graph G find the smallest set $S \subseteq V(G)$ such that every node of G is either in S or has a neighbor in S. This problem is fpt parameterized by the treewidth of the graph. Give an algorithm that solves the problem given a tree decomposition of width w in time $O(9^w \cdot w \cdot n)$.

Task H9   (5 credits)

The TRIANGLE PACKING problem is defined as follows: given a graph $G = (V, E)$ and an integer k, decide whether G has k vertex-disjoint 3-cycles. Use the idea of randomly coloring the vertices of G with k colors to enable easy detection of vertex-disjoint triangles. What is the running time of your algorithm? And what is the failure probability?

Task H10   (10 credits)

Design a fixed parameter algorithm for finding an $k \times k$-grid as a *minor* in a graph. The parameter is k.
Hint: Design first an algorithm which says whether there exists a $k \times k$-grid as minor.