Parameterized Algorithms WS 2021
Prof. Dr. P. Rossmanith
Dr. E. Burjons, M. Gehnen, H. Lotze, D. Mock

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Exercise Sheet 02

Task T4

Give a polynomial kernel for the following problem.

Input:	A sequence of marbles, each with a non-negative integer weight and color.
Parameter:	A positive integer k.
Question:	Can we remove marbles of total weight at most k, such that for each color, all marbles of that color are consecutive?

Task T5

We consider the kernel for Vertex Cover by Nemhauser/Trotter. Consider a cycle G with n vertices.

a) How does G_B look like?

b) What are the sets V_0 and C_0?

c) How good is the problem reduction in this case?

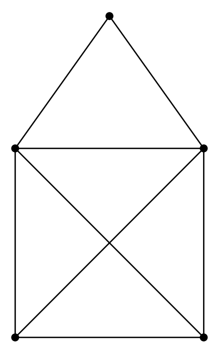
Task T6

Show that planarity is a hereditary property. Is the forbidden set finite or infinite? If your answer is "finite" then construct the forbidden set; if your answer is "infinite", then construct an infinite family $\mathcal{F}$ of non-planar graphs such that

- for all $G \in \mathcal{F}$, all proper subgraphs of G are planar;
- for all distinct $G_1, G_2 \in \mathcal{F}$, we have that G_1 is not an induced subgraph of G_2.

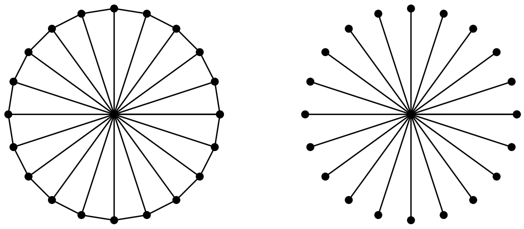
Task T7

Does the problem reduction help for the following graph (*Haus des Nikolaus*)? (As in exercise T5)



Task H3 (5 credits)

How does the Nemhauser/Trotter kernel look like for the *wheel* and the *star*?



Task H4 (5 credits)

Let Π be a decidable parameterized problem. Show that Π is fixed-parameter tractable if and only if there exists a kernelization algorithm for it.

Task H5 (5 credits)

Consider the Planar Independent Set problem: Given a planar graph G and an integer k, decide whether G has an independent set of size at least k. Design an algorithm that takes as input (G, k), where G is planar, and outputs in polynomial time an equivalent instance (G', k') such that $|V(G')| = O(k)$.