RWTH Aachen **WS 2014**
Lehrgebiet Theoretische Informatik Exam 1
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Parameterized Algorithms Exam

Partial solutions will be credited, so do not give up if you cannot provide the complete solution.

Exercise 1 (10 Points)

Consider the following problem: The input is a graph $G = (V, E)$ and an integer k; the question is whether one can delete at most k vertices such that the resulting graph is 4-regular. The parameter is the integer k. Show that this problem is in FPT by designing a branch-and-bound algorithm for it.

Exercise 2 (5 + 5 Points)

We want to show that the following problem has a randomized FPT algorithm.

Input:	A graph $G = (V, E)$ and an integer k.
Parameter:	The integer k.
Question:	Does G contain at least k edge-disjoint cycles of length at most 5?

1. Use color-coding to design a randomized FPT algorithm for this problem.
2. Analyze the success probability of your color-coding strategy and the running time of your algorithm.

Exercise 3 (10 Points)

Consider the following variant of the Max Sat problem: you are given a formula φ in CNF and an integer parameter k; the question is to decide whether there exists as assignment to the variables of φ such that the number of clauses with *exactly* one true literal is at least k.

Show that this problem is W[1]-hard. [**Hint:** Reduce from Independent Set.]

Exercise 4 (10 Points)

Given a graph $G = (V, E)$, an *induced matching* of G is a set of edges $F \subseteq E$, such that the edge set of the induced subgraph $G[V(F)]$ is F itself. The *size* of an induced matching is the number of edges in it. The Induced Matching problem is given a graph G and an integer k, to decide whether G has an induced matching of size at least k.

Design a linear kernel for this problem on graphs of maximum degree d, where d is a constant.