**RWTH Aachen** **WS 2018**
**Lehrgebiet Theoretische Informatik** Problem Set 12
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# Parameterized Algorithms Tutorial

In the lecture characterized graph classes with bounded expansion using centered colorings. A *centered coloring* is a coloring of a graph such that every connected subgraph has a color that is used exactly once. A graph class $\mathcal{G}$ has *bounded expansion* if for every $r \in \mathbb{N}$ there exists $f(r)$ such that for every $G \in \mathcal{G}$ there exists a coloring with at most $f(r)$ colors where every subgraph with at most $r$ colors has a centered coloring.

## Tutorial Exercise T1

The tree-depth of a graph $G$ may be defined as the minimum height of a forest $F$ with the property that every edge of $G$ connects a pair of nodes that have an ancestor-descendant relationship to each other in $F$. Show that a graph has a centered coloring with at most $d$ colors if and only if it has treedepth at most $d$.

## Tutorial Exercise T2

We consider robber-cops games with the additional constraint that once a cop is placed at a certain location it cannot leave the location. Show that a graph $G$ has treedepth at most $d$ if and only if $d$ cops can catch a robber in $G$.

## Remark

One can obtain an alternative definition of bounded expansion:
$\mathcal{G}$ has bounded expansion if for every $r \in \mathbb{N}$ there exists $f(r)$ such that for every $G \in \mathcal{G}$ there exists a coloring with at most $f(r)$ colors where every subgraph with at most $r$ colors has treedepth at most $r$.

## Homework H1

Show that the treewidth of a graph is smaller than the treedepth of a graph.

## Homework H2

Show that a path of length $2^d$ has treedepth at most $O(d)$.

## Homework H3

Show that a graph with treedepth $d$ may contain paths of length at most $2^{O(d)}$.