RWTH Aachen
Theoretical Computer Science
Rossmanith–Gehnen

WS 2025
Exercise 11
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Exercise for Analysis of Algorithms

Exercise 43

In this exercise we consider the following (regular) CFG G:

$$S \to abA \mid bS \mid a$$
$$A \to bA \mid aS$$

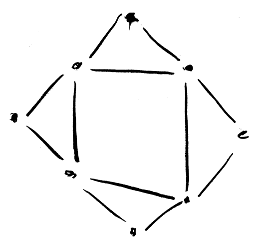
1. Find a generating function for number of words s_n in $L(G)$ that have length n.
2. What is the dominant singularity and what kind of singularity is it?
3. What is the exponential growth of s_n?
4. How precisely can you estimate s_n with just the knowledge of the dominating singularity and its nature?
5. Find a closed formula for s_n with an additive error of at most $O(0.8^n)$.

Exercise 44

An algorithm I computes an optimal independent set for an undirected graph $G = (V, E)$ of size n as follows: It picks a vertex v with maximal degree. If this degree is at most two, then the graph is a collection of cycles and paths and the solution is computed in linear time.

Otherwise, the optimal independent set either contains v (and then cannot contain any vertex in $N(v)$) or it does not. Hence, the algorithm recursively computes the two independent sets $I(G[V - N(v)])$ and $I(G[V - \{v\}])$ and then chooses the bigger one, or the first if they have the same size.

1. Simulate the algorithm on this graph:



2. Estimate its asymptotic running time up to a constant factor.

Exercise 45
Prove that

$$[z^n](1-z)^w \sim \frac{n^{-w-1}}{\Gamma(-w)}$$

for $w \in \mathbf{C}$ without using the theorem of the lecture. (The idea of this assignment is to get a deeper insight into the theorem.)
Hint: Use Newton's formula. Now replace the binomial coefficient by factorials or the gamma function. In the first case, you need to be careful with a definition of factorials for real numbers. In general, however, $\Gamma(n+1) = n!$.

Exercise 46
Approximate $[z^n]\frac{1}{2-e^z}$ up to an error of $O(12^{-n})$.

Exercise 47
Determine g_n up to an additive error of $O(4^n)$ for

$$G(z) = \sum_{n=0}^{\infty} g_n z^n = \frac{15z^2+8z+1}{15z^2-8z+1}.$$