

Old Exam (2014) 0

This is an old exam from 2014.

Task T1

Consider the following algorithm for searching an array $a[1, \dots, n]$ for an element x . We assume that the array is sorted in increasing order and that the element x is at some random location in the array. Let B_n be the expected number of comparisons on an n -element array. Write down a recurrence for B_n . What is B_3 ?

Algorithm: Binary Search with randomly chosen pivot element

1. Choose randomly and with uniform probability an $i \in \{1, \dots, n\}$.
2. If $a[i] = x$, output i and halt.
3. Continue recursively on the left subarray, if $x < a[i]$, or the right subarray, if $x > a[i]$.

Task T2

An alphabet Σ consists of two numeric characters 1, 2 and four alphabetic characters a, b, c, d . Find and solve a recurrence relation for the number of words of length n in Σ^* , where there are no consecutive (identical or distinct) numeric characters.

Task T3

Find an expression for

$$[z^n] \frac{1}{(1-z)^2} \ln \frac{1}{1-z}.$$

Your solution can include a sum!

Task T4

Sort the series with the following generating functions by their asymptotic growth. Justify your steps!

1. $A(z) = \frac{1}{\sqrt{2-\frac{1}{z}}}$.
2. $B(z) = \frac{z}{2-3z+z^2}$.
3. $C(z) = \frac{e^{-z-z^2/2}}{1-z}$.