Analysis of Algorithms WS 2022
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Date: January 16, 2023

Exercise Sheet 10

Due date: next tutorial session, preferably in groups

Tutorial Exercise T10.1

Find the exponential growth of the following functions:

a) $2^n n^3$

b) $\left(\frac{2}{3}\right)^{2n} + 5$

c) $[z^n]\frac{1}{\sqrt{1-5z}}$

d) $[z^n]\frac{z^2-1}{(z-1)(z-5)}$

e) $[z^n]\frac{1}{e-e^{3/2-z^2}}$

Tutorial Exercise T10.2

Find very large and very small functions with exponential growth 1, 0 and ∞.

Tutorial Exercise T10.3

Sort the following generating functions *within one minute* by their exponential growth!

1. $A(z) = \frac{1}{\sqrt{1-z/2}}$
2. $B(z) = \frac{1}{1-e^{z-1/3}}$
3. $C(z) = \frac{1+z}{1-z}$

Tutorial Exercise T10.4

Daniel lives in an old German building *(Altbau)* and has one of these typical guest bathrooms which are narrow but very long. He wants to improve its outdated look by paving the floor with new tiles. Luckily, he found a bunch of tiles which fell from the back of a truck. Since the tiles are in a specific color pattern he does not know how he can pave this strip so that it looks best. The tiles are (identical) 1×2 tiles he can use and rotate by 90 degrees: ▯. His approach is to try every tiling by laying it out and to give it a beauty-value. One such tiling needs $O(n)$ time. He thinks that, if the exponential growth of the total time needed is less than 4.5^n, he can find the most beautiful bathroom tiling without loosing his mind. Can he find it or is he doomed?

Homework Exercise H10.1

Daniel found somewhere else a new tile shape, namely an 1×1 tile: □ . This increases the options and the time needed for him to try out all possibilites. Can he still find the best looking option in time if he also considers the new tile?

Homework Exercise H10.2

In this exercise we will look at 2-3-trees. They are rooted, ordered trees. Each internal node has either two or three children. As usual, the size of a 2-3-tree will be the number of its internal nodes.

1. How can you define 2-3-trees recursively?
2. Enumerate all 2-3-trees of size two. How many are there? How many trees exist of sizes zero and one?
3. Find a generating function $Q(z)$ for the number q_n of 2-3-trees with size n.
4. What is the dominant singularity of $Q(z)$ and what is the exponential growth of q_n? Use a computer algebra system. Do not give up when you see horrifying formulas.