Analysis of Algorithms WS 2022
Prof. Dr. P. Rossmanith
M. Gehnen, H. Lotze, D. Mock

Date: December 12, 2022

Exercise Sheet 07

Tutorial Exercise T7.1

Compute the generating functions of the following series:

(a) $a_n = 2^n + 3^n$ (b) $b_n = (n+1)2^{n+1}$ (c) $c_n = \alpha^n \binom{k}{n}$
(d) $d_n = n - 1$ (e) $e_n = (n+1)^2$

Tutorial Exercise T7.2

Compute:

(a) $[z^n]\dfrac{1}{1+2z}$ (b) $[z^n]\dfrac{z+1}{z-1}$ (c) $[z^n]\left(\dfrac{z+1}{z-1}\right)^2$ (d) $[z^n]\dfrac{1}{\sqrt[3]{5+z}}$

Homework Exercise H7.1

Let $A(z)$ and $B(z)$ be the OGFs of two series a_n and b_n.
The convolution $c_n = (a_n)_{n=0}^{\infty} * (b_n)_{n=0}^{\infty}$ of a_n and b_n is defined as

$$c_n = \sum_{k=0}^{n} a_k b_{n-k}.$$

For example,

$$(n)_{n=0}^{\infty} * (3^n)_{n=0}^{\infty} = \left(\sum_{k=0}^{n} k3^{n-k}\right)_{n=0}^{\infty}.$$

Prove that the OGF of the convolution of a_n and b_n is $A(z)B(z)$.

Homework Exercise H7.2

Solve this recurrence using generating functions:

$$a_n = 2a_{n-1} + 3a_{n-2}$$

and $a_0 = 0$, $a_1 = 2$.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1