Analysis of Algorithms WS 2022
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Exercise Sheet 05

Due date: next tutorial session

Tutorial Exercise T5.1
Let $x \in \mathbf{R}^+$. Is $\lceil \sqrt{x} \rceil = \left\lceil \sqrt{\lceil x \rceil} \right\rceil$?

Tutorial Exercise T5.2
Solve the following recurrence relation by order reduction:

$$a_0 = 8000 \qquad a_1 = \frac{1}{2} \qquad a_{n+2} + a_{n+1} - n^2 a_n = n!$$

Tutorial Exercise T5.3
Solve the following recurrence relation:

$$a_n = n + 1 + \frac{1}{n} \sum_{k=0}^{n-1} a_k \text{ for } n > 0 \text{ and } a_0 = 2$$

Homework Exercise H5.1
Solve the following recurrence relation by order reduction:

$$a_0 = 0 \qquad a_1 = 1 \qquad a_{n+2} + a_{n+1} - n^2 a_n = n!$$

This is the same recurrence relation as in T2, but the initial conditions are different. Whereas that exercise asked for a solution using order reduction, for this exercise you can choose whatever method you like.

Homework Exercise H5.2
How often is the loop in the following excerpt executed if $0 < i$ holds at the beginning?

```
while i <= j
    i := i+j;
    j :=j+10;
```

Homework Exercise H5.3
Our task is to generate a word of length n over the alphabet $\{0, 1\}$, which contains neither two consecutive zeros nor three consecutive ones.
Daniel proposes the following algorithm: The algorithm generates a word of length n uniformly at random. If the word fulfills the property, it is returned. Otherwise, the algorithm tries again until it finds one.
What is the expected number of rounds the algorithm needs depending on n? It is enough to give the fastest growing term, i.e., something of the form $\alpha f(n) + o(f(n))$. Also, give the expected number of rounds for 32bit words explicitly.

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