Analysis of Algorithms WS 2022 Prof. Dr. P. Rossmanith M. Gehnen, H. Lotze, D. Mock



Date: November 7, 2022

Exercise Sheet 02

Due date: next tutorial session

Tutorial Exercise T2.1

Let S_N be the expected number of pushs on the stack in the Quicksort program, where the input consists of a random permutation of N distinct keys. Analyze S_N for all values of N and M.

Tutorial Exercise T2.2

The next program is presented in x86 assembler language: Again the array ds[0]...ds[2*N-2] contains N pairwise distince natural numbers. Each permutation occurs with the same probability. How often is each instruction of this program executed on average?

maxElem:	mov	ax, OxFFFF	$A \ ax \leftarrow -1;$
	xor	dx, dx	$A \ dx \leftarrow 0;$
next:	cmp	dx, N	$B \ i < N \ ?$
	jae	done	B jump if above or equal $(i \ge N)$
	mov	bx, ds:[2*dx]	$C \ bx \leftarrow a[dx]$
	cmp	bx, max	C bx > max?
	jna	skip	C jump if not above $(bx \leq N)$
	mov	ax, bx	$D \ ax \leftarrow bx$
skip:	add	dx, 0x0002	$E ax \leftarrow ax + 1;$
	jmp	next	E jump
done:	push	ax	${\cal F}$ push the maximum on the stack

Homework Exercise H2.1

We already analyzed C_n , the *total* expected number of comparisons in the two innermost whileloops of the quicksort algorithm (see the program fragment below).

What is the expected number of executions of the single comparison a[i] < k?

```
[...]
    i = l - 1; j = r ; k = a[j];
    do{
        do{i++;} while ( a[i] < k );
        do{j--;} while ( k < a[j] );
        t = a[i]; a[i] = a[j]; a[j] = t;
        } while ( i < j );
[...]</pre>
```

Homework Exercise H2.2

We consider the following Algorithm. The array **a** contains a random permutation of the numbers $1, \ldots, N$.

```
void doSomething(int *a, int N)
{
    int i;
    for (i=0; i<N-1; i++) /* 1 */
    while (a[i] > a[i+1]) /* 2 */
        a[i]--; /* 3 */
}
```

How often is line 3 executed on average?