Analysis of Algorithms, WS 2020
Prof. Dr. P. Rossmanith
Dr. E. Burjons, H. Lotze, D. Mock

Date: 20 January 2021

Exercise Sheet 10

Problem T22

Find the exponential growth of the following functions:

a) $2^n n^3$

b) $\left(\frac{2}{3}\right)^{2n} + 5$

c) $[z^n]\frac{1}{\sqrt{1-5z}}$

d) $[z^n]\frac{z^2-1}{(z-1)(z-5)}$

e) $[z^n]\frac{1}{e-e^{3/2-z^2}}$

Problem T23

Find very large and very small functions with exponential growth 1, 0 and ∞.

Problem T24

Sort the following generating functions *within one minute* by their exponential growth!

1. $A(z) = \frac{1}{\sqrt{1-z/2}}$
2. $B(z) = \frac{1}{1-e^{z-1/3}}$
3. $C(z) = \frac{1+z}{1-z}$

Problem H23 (5 credits)

Prove that

$$\binom{-r}{n} = (-1)^n \binom{r+n-1}{n}$$

for $r \in \mathbf{R}, n \in \mathbf{Z}$.

Problem H24 (10 credits)

Prove that

$$[z^n](1-z)^w \sim \frac{n^{-w-1}}{\Gamma(-w)}$$

for $w \in \mathbf{C}$ without using the theorem of the lecture. (The idea of this assignment is to get a deeper insight into the theorem.)

Hint: Use Newton's formula, then the first exercise on this sheet. Now replace the binomial coefficient by factorials or the gamma function. In the first case, you need to be careful with a definition of factorials for real numbers. In general, however, $\Gamma(n+1) = n!$.

Problem H25 (10 credits)

In this exercise we will look at 2-3-trees. They are rooted, ordered trees. Each internal node has either two or three children. As usual, the size of a 2-3-tree will be the number of its internal nodes.

1. How can you define 2-3-trees recursively?
2. Enumerate all 2-3-trees of size two. How many are there? How many trees exist of sizes zero and one?

3. Find a generating function $Q(z)$ for the number q_n of 2-3-trees with size n.

4. What is the dominant singularity of $Q(z)$ and what is the exponential growth of q_n? Use a computer algebra system. Do not give up when you see horrifying formulas.