Analysis of Algorithms, WS 2020
Prof. Dr. P. Rossmanith
Dr. E. Burjons, H. Lotze, D. Mock

Date: 13 January 2021

Exercise Sheet 09

If $\int_1^n |f^{(i)}(x)|\,dx$ exists for $1 \le i \le 2m$, then

$$\sum_{k=1}^{n} f(k) = \int_1^n f(x)\,dx + \frac{1}{2}f(n) + C + \sum_{k=1}^{m} \frac{B_{2k}}{(2k)!} f^{(2k-1)}(n) + R_m,$$

where $R_m = O\left(\int_n^\infty |f^{(2m)}(x)|\,dx\right)$ and $B_k = n![z^n]z/(e^z-1)$ are the Bernoulli-numbers:

n	0	1	2	3	4	5	6
B_n	1	$-\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{30}$	0	$\frac{1}{42}$

Problem T20
Approximate the following sum up to an error of $O(n^{-5})$:

$$\sum_{k=1}^{n} \frac{1}{k^2}$$

Find the constant C in Euler's summation formula by looking up $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Test your result for $n = 1000$. Use your favorite computing software.

Problem T21
If you use Euler summation on a polynomial function, can you get an *exact* solution? Prove it or find a counterexample.

Problem H21 (10 credits)
Approximate the following sum up to an error of $O(n^{-5})$:

$$\sum_{k=1}^{n} \frac{1}{k^{5/2}}$$

Problem H22 (10 credits)
Find a function $f(n)$ in closed form such that

$$\prod_{k=1}^{n} k^k = f(n)(1 + O(1/n^2)).$$

Use Euler summation. It is okay if you cannot find the correct constant in the sum.