RWTH Aachen **SS 2014**
Lehrgebiet Theoretische Informatik Problem Set 3
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Analysis of Algorithms

Problem 3-1

In this exercise, we consider Prim's Algorithm, which computes a minimum spanning tree. The input to this algorithm is a graph $G = (V, E)$, a weight function on the edges $w : E \to \mathbf{R}$ and a starting node r.

```
 1  for each u ∈ V do
 2      key[u] ← ∞
 3      π[u] ← NIL
 4  key[r] ← 0
 5  M ← V
 6  while (M ≠ ∅) do
 7      u ← min-from(M)
 8      for each v ∈ neighbors(u) do
 9          if (v ∈ M) ∧ (w(u, v) < key[v]) then
10              π[v] ← u
11              key[v] ← w(u, v)
```

Construct the control flow graph, a spanning tree in the control flow graph, the fundamental cycles, a corresponding linear system of equations and a solution to this system.

Exercise 3-2 The following program prints a lot of numbers. How many?

```
void snapl(int n) {
 if ( n == 0 || n == 2 ) return;
  if ( n == 1 ) std::cout << 42 << std::endl;
  else {
    snapl( n - 2 );
    snapl( n - 3 );
    snapl( n - 2 );
    snapl( n - 3 );
    snapl( n - 2 );
  }
}
```

Homework Assignment 3-1 (10 Points)

Consider the following program: The input to this program is an array $a[0, \ldots, n-1]$ that contains n pairwise distinct integer keys in random order.

a) Explain how this program sorts the given array.

b) Analyse how often each instruction of the program is executed on average depending on n.

```
int sel_sort ( int a[], int n ) {
  for ( int i = 0; i < n; ++i ) {
    int min = i;
    for ( int j = i; j < n; ++j ) {
      if ( a[j] < a[min] ) {
        min = j;
      }
    }
    int temp = a[i];
    a[i] = a[min];
    a[min] = temp;
  }
}
```

c) There is only one instruction whose analysis is not trivial. Which one is it?

Make a table for small values of n by hand that lists the results for this instruction. Compare the table entries with the results from your closed formula that you obtained in b).

Homework Assignment 3-2 (10 Points)

Solve the following recurrence: Let $b_1 = b_2 = b_3 = 1$ and

$$b_n = 3b_{n-1} - 4b_{n-2} + 12b_{n-3} \text{ for } n > 3.$$